



## NOTES

### Rectangular Coordinates

$$\nabla f = \hat{x} \frac{\partial f}{\partial x} + \hat{y} \frac{\partial f}{\partial y} + \hat{z} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \nabla \cdot \nabla f$$

### Cylindrical Coordinates

$$\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial f}{\partial \phi} + \hat{z} \frac{\partial f}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{r} \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \left( \frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right)$$

$$= \begin{vmatrix} \hat{r} \frac{1}{r} & \hat{\phi} & \hat{z} \frac{1}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix}$$

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

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### Spherical Coordinates

$$\nabla f = \hat{r} \frac{\partial f}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial f}{\partial \theta} + \hat{\phi} \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \hat{r} \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{r} \left( \frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r} (r A_\phi) \right) + \hat{\phi} \frac{1}{r} \left( \frac{\partial}{\partial r} (r A_\theta) - \frac{\partial A_r}{\partial \theta} \right)$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$$

### General Curvilinear Coordinates

$$\nabla f = \hat{x}_1 \frac{1}{h_1} \frac{\partial f}{\partial x_1} + \hat{x}_2 \frac{1}{h_2} \frac{\partial f}{\partial x_2} + \hat{x}_3 \frac{1}{h_3} \frac{\partial f}{\partial x_3}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{h_1 h_2 h_3} \left( \frac{\partial}{\partial x_1} h_2 h_3 A_1 + \frac{\partial}{\partial x_2} h_3 h_1 A_2 + \frac{\partial}{\partial x_3} h_1 h_2 A_3 \right)$$

$$\nabla \times \mathbf{A} = \hat{x}_1 \frac{1}{h_2 h_3} \left[ \frac{\partial}{\partial x_2} h_3 A_3 - \frac{\partial}{\partial x_3} h_2 A_2 \right] + \hat{x}_2 \frac{1}{h_3 h_1} \left( \frac{\partial}{\partial x_3} h_1 A_1 - \frac{\partial}{\partial x_1} h_3 A_3 \right) + \hat{x}_3 \frac{1}{h_1 h_2} \left( \frac{\partial}{\partial x_1} h_2 A_2 - \frac{\partial}{\partial x_2} h_1 A_1 \right)$$

$$\nabla^2 f = \frac{1}{h_1 h_2 h_3} \left[ \frac{\partial}{\partial x_1} \left( \frac{h_2 h_3}{h_1} \frac{\partial f}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left( \frac{h_3 h_1}{h_2} \frac{\partial f}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left( \frac{h_1 h_2}{h_3} \frac{\partial f}{\partial x_3} \right) \right]$$

$$h_1 \quad h_2 \quad h_3 \quad x_1 \quad x_2 \quad x_3$$

Rectangular  
Cylindrical  
Spherical

1	1	1	x	y	z
1	r	1	r	φ	z
1	r	r sin θ	r	θ	φ