CRITERIA FOR GLOBAL MAX, MIN.

f has a global max on a given domain I, if there is a point p in I such that f(p) is at least as great as the value of f at any other point of I.
A given f on a given I may or may not have a global max.

1) The only possible place a function could have a global max (or min) would be at a critical point or an endpoint, so if you are looking for them you only have to look at those points.

2) If the function f is continuous on I, and I is a *closed bounded* interval, then f always has both a global max and a global min on I, so you just find the values of f at all critical points in I, and at the endpoints of I, and see which value is biggest (or smallest).

3) If the function is continuous on I, and I is an open interval, then f may not have global extrema, but if it has them they must occur at critical points. Thus if f has no critical points on I, then f has no global extrema on I. If f does have critical points on I, you must look further to discover whether they do or do not include some global extrema.

4) If f is continuous on I and I is an open interval, and if the limits of f at both endpoints are equal to +‘, then f does have a global min on I, but not a global max. In particular if f has limit +‘ at both endpoints, and if f has exactly one critical point in I, then that critical point must be the global min. If more generally it has limit +‘ at both endpoints, then the critical point where f has smallest value is the global min. Similarly, if f has limit -‘ at both endpoints of I, then f has a global max (but not a global min) on I, and it must occur at the critical point in I where f has largest value.

5) Suppose f is continuous on I = (a,b), an open interval, and suppose that c is the smallest critical number of f in I, i.e. the one closest to a, and that d is the largest critical number of f, i.e. the one closest to b. Thus we have $a < c < d < b$, and there are no critical numbers of f in either interval (a,c) or (d,b). If also f is increasing on (a,c) and decreasing on (d,b), then f has a global max on I, and it occurs at the critical number where f has the greatest value. If on the other hand (you guessed it) f is decreasing on (a,c) and increasing on (d,b), then f has a global min on I, at the critical point where f has the smallest value.

(Note in criterion 5, the point is that f has a global min. say, on the closed interval [c,d], so if f is decreasing on (a,c) and increasing on (d,b), then the min for the interval [c,d] is also a min for the whole interval (a,b). So the idea is to reduce to the case of a closed interval.)

6) Here is another cute one: if f is continuous and differentiable everywhere on I, an open interval, and if f is concave up everywhere on I, and if f has exactly one critical number on I, then f has a global min at that critical number, (but no global max anywhere). (Try to picture it.)

The whole point to understanding these criteria is pretty much that f is always either increasing or decreasing on any interval which does not contain critical points, and then you just picture that and look at the graph.