Could GR Contextuality Resolve the Missing Mass Problem?

W.M. Stuckey[1], Timothy McDevitt[2], A.K. Sten[3] and Michael Silberstein[4]

[1] Department of Physics
Elizabethtown College
Elizabethtown, PA 17022
stuckeym@etown.edu
corresponding author
[2] Department of Mathematical Sciences
Elizabethtown College
Elizabethtown, PA 17022
mcdevittt@etown.edu
[3] Department of Physics
University of North Texas
Denton, TX 76201
AlexanderSten@my.unt.edu
[4] Department of Philosophy and Foundations of Physics
Committee for Philosophy and the Sciences
University of Maryland
College Park, MD 20742
msilbers@umd.edu

Abstract. In Newtonian gravity, mass is an intrinsic property of matter while in general relativity (GR), mass is a contextual property of matter, e.g., when two different GR spacetimes are adjoined. Herein, we explore the possibility that the astrophysical missing mass attributed to non-baryonic dark matter (DM) actually obtains because we have been assuming the Newtonian intrinsic view of mass rather than the GR contextual view. Perhaps, we should model astrophysical phenomena via combined GR spacetimes to better account for their complexity. Accordingly, we consider a GR ansatz in fitting galactic rotation curve data (THINGS), X-ray cluster mass profile data (ROSAT/ASCA), and CMB angular power spectrum data (Planck 2015) without DM. We find that our fits compare well with both modified gravity programs and DM programs.

Essay written for the Gravity Research Foundation 2018 Awards for Essays on Gravitation
Submitted 16 March 2018

1. Introduction

The astrophysical missing mass problem is unresolved despite decades of work on modified gravity programs, dark matter theory, and experimental searches for non-baryonic dark matter (DM). Due to the complexity of "realistic distributions of matter in galaxies, we have neither analytic, nor numerical solutions to general relativity (GR) from which orbits can be predicted" [1]. Indeed, it is not reasonable to expect an exact GR solution applicable to most missing mass phenomena. So, perhaps it is reasonable to consider GR deviations from Newtonian gravity to account for missing mass phenomena, even in such weak gravitational fields. In fact, Cooperstock et al. used GR instead of Newtonian gravity in fitting galactic rotation curves (RC's) and found that the non-luminous matter in galaxies "is considerably more modest in extent than the DM extent claimed on the basis of Newtonian gravitational dynamics" [2–4]. Cooperstock also showed [5] that there is no Newtonian limit for the flat rotation curves of GR's axially-symmetric van Stockum solution. Herein, we propose another mechanism by which GR deviates from Newtonian gravity in order to account for missing mass phenomena without DM.

The difference we are pointing out is the well-known result per GR that matter can simultaneously possess different values of mass when it is responsible for different combined spatiotemporal geometries. This spatiotemporal contextuality of mass is not present in Newtonian gravity where mass is an intrinsic property of matter. For example, when a Schwarzschild vacuum surrounds a spherical matter distribution the "proper mass" M_p of the matter, as measured locally in the matter, can be different than the "dynamic mass‡" M in the Schwarzschild metric responsible for orbital kinematics about the matter [6]. This difference is attributed to binding energy and goes as $dM_p = \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1/2} dM$, which means it is too small to account for missing mass phenomena, but it is evidence of GR contextuality for mass. In another example, suppose a Schwarzschild vacuum surrounds a sphere of Friedmann-Lemaître-Robertson-Walker (FLRW) dust connected at the instanteously null Schwarzschild radial coordinate. The dynamic mass M of the surrounding Schwarzschild metric is related to the proper mass M_p of the FLRW dust by [7]

$$\frac{M_p}{M} = \begin{cases} 1 & \text{flat model} \\ \dfrac{3\left(\eta - \sin(\eta)\right)}{4\sin^3(\eta/2)} \geq 1 & \text{closed model} \\ \dfrac{3\left(\sinh(\eta) - \eta\right)}{4\sinh^3(\eta/2)} \leq 1 & \text{open model} \end{cases} \tag{1}$$

where η is conformal time. Using this well-known embedding for the closed FLRW model (used originally to model stellar collapse [8]), we have for the ratio $\frac{M_p}{M}$ of a ball

‡ Typically, "dynamical mass" and "luminous mass" are the terms used with dynamical mass larger than luminous mass. Our terminology is following the GR convention.

of FLRW closed-model dust surrounded by Schwarzschild vacuum as joined at FLRW radial coordinate χ_o

$$\frac{M_p}{M} = \frac{3(2\chi_o - \sin(2\chi_o))}{4\sin^3(\chi_o)} \tag{2}$$

(Figure 1), where

$$ds^2 = -c^2 d\tau^2 + a^2(\tau)\left(d\chi^2 + \sin^2\chi d\Omega^2\right) \tag{3}$$

is the closed FLRW metric. The heuristic picture here is that a ball of FLRW dust has collapsed from its expanding FLRW cosmological context leaving a vacuum region about the ball. The dynamic mass of the collapsed ball of dust measured by a vacuum region observer in orbit about the collapsed ball is then less than the proper mass of the ball as determined by observers in the remaining expanding FLRW global context. Since it is dynamic mass that is used for mass-luminosity ratios, there will be a discrepancy between dynamic mass as determined by mass-luminosity ratios and larger scale determinations of proper mass. And, this discrepancy can be quite large. For $\chi_o = \frac{\pi}{2}$ Eq(2) gives $\frac{M_p}{M} = 2.36$ and for $\chi_o = 0.8\pi$ Eq(2) gives $\frac{M_p}{M} = 22.1$. The ratio quickly increases beyond $\chi_o = \frac{\pi}{2}$ because $dM < 0$, since the spherical area starts decreasing with increasing χ. It is also the case that the extrinsic curvature of the interface changes sign for $\chi_o > \frac{\pi}{2}$, so this region has the dust surrounding the vacuum and is not what we're considering, but we're not proposing that this is an exact GR solution explaining away DM. Again, no such exact solution is likely forthcoming, given the complexity of matter distribution in most astrophysical situations. Rather, we're simply pointing out that the GR contextuality of mass created when different GR spacetimes are combined allows for large differences between proper and dynamic mass. Essentially, mass is an intrinsic property of matter per Newtonian gravity, but mass is a geometric consequence of matter per GR. Since two different spacetime geometries may be associated with one and the same matter in a combined GR solution, mass is a contextual property of matter per GR. So, given the complexity of most astrophysical matter distributions, combined GR spacetimes might provide a better model of dark matter phenomena than Newtonian gravity or a single GR spacetime. We should quickly point out that this may prima facie seem to constitute a violation of the equivalence principle, as understood to mean inertial mass equals gravitational mass, since inertial mass can't be equal to two different values of gravitational mass. But, the equivalence principle says simply that spacetime is locally flat [9] and that is certainly not being violated here nor with any solution to Einstein's equations. So, we believe it is reasonable to consider this difference between GR and Newtonian gravity as the source of missing mass phenomena§.

§ We previously considered contextuality motivated by disordered locality requiring a modified gravity [10], but here we consider the contextuality already inherent in combined GR spacetimes.

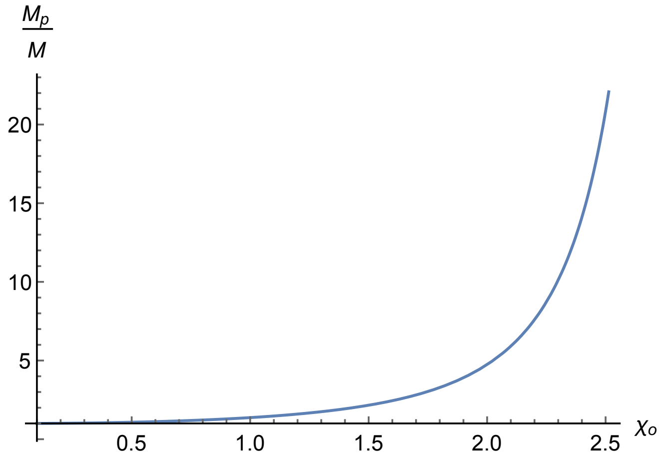

Figure 1. Plot of $\frac{M_p}{M}$ as a function of the radial coordinate χ_o at the junction between the closed, matter-dominated FLRW spacetime and the Schwarzschild spacetime. In GR, unlike Newtonian gravity, matter can simultaneously have two different values of mass. Thus, mass is not an intrinsic property of matter in GR, it is contextual.

2. The Ansatz

While proper mass M_p must be ~ 10 times larger than dynamic mass M in order to account for missing mass phenomena, the correction to the background spacetime metric responsible for missing mass phenomena is small. For example, assuming circular orbits as is common for fitting galactic rotation curves, we have $v^2 r = GM_p(r)$ where $M_p(r)$ is the (larger) proper mass inside the circular orbit at radius r and v is the orbital speed. This gives $\dfrac{2GM_p(r)}{c^2 r}$ of the Schwarzschild metric equal to $2\dfrac{v^2}{c^2}$. The largest galactic rotation speeds are typically only $10^{-3}c$, so the metric deviation from flat spacetime per the Schwarzschild metric is $\sim 10^{-6}$, an empirically small metric correction for galactic kinematics. Intracluster medium gas is even more rarefied and the potentials used for FLRW metric perturbations leading to anisotropies in the angular power spectrum of the cosmic microwave background (CMB) are already $\ll 1$ to include DM.

In this geometric view, missing mass phenomena are understood as small deviations from some background metric. Of course, if one favors an exotic new kind of matter a la DM to account for missing mass phenomena, then it is important to note that the exotic new matter is far more prevalent than ordinary matter. But in our approach, it is more important to note that we're dealing with weak gravitational fields. It is customary to expect deviations of GR from Newtonian gravity for a strong gravitational field with its large spacetime curvature, but attempting to account for missing mass phenomena via GR actually requires that GR deviate from Newtonian gravity for weak fields, as we showed above. Thus, contrary to conventional thinking, what we're advocating is a geometric view of even weak gravitational fields, at least when the matter is distributed on astronomical scales. Of course, this is not unprecedented.

In addition to Cooperstock et al. noted above, Moffat & Rahvar used "The weak field approximation of MOG (modified gravity)" as a perturbation of "the metric and the fields around Minkowski space-time" in fitting galactic RC's [11] and X-ray cluster mass profiles [12] without DM. Certainly modified Newtonian dynamics (MOND) can be viewed in this fashion, since MOND advocates an extremely small modification to Newtonian acceleration on astronomical scales in the context of flat spacetime (Newtonian gravity), and acceleration due to gravity in flat spacetime is replaced by curved spacetime in GR. So, MOND's premise is similar to ours, i.e., a small change in spacetime curvature on astrophysical scales (equivalent to a small change to acceleration in Newtonian gravity) replaces the need for a greatly increased mass in accounting for galactic dynamics. Again, a correction is justified if the contextual nature of mass in combined GR spacetimes provides a better model of dark matter phenomena than Newtonian gravity or a single GR spacetime, and this seems a reasonable assumption given the complexity of most astrophysical matter distributions.

Since

- $M = M_p$ for the spatially flat FLRW model surrounded by Schwarzschild vacuum per Eq(1).
- ${}^{(3)}R = 0$ for both the spatially flat FLRW dust and Schwarzschild vacuum.
- We're talking about weak gravitational fields.

we might attribute the large mass difference between M and M_p to a small difference in *spatial* curvature. Indeed, the difference between M_p and M shown in Eq(2) obtains from the integrated difference in spatial geometry between the closed, matter-dominated FLRW dust ball and the surrounding Schwarzschild vacuum [7]

$$\frac{M_p}{M} = \frac{\int_0^{\chi_o} \sin^2\chi \, d\chi}{\int_0^{\chi_o} \sin^2\chi \, d(\sin\chi)} \tag{4}$$

In the case of dark energy as pertains to the SCP Union2.1 SN Ia data, we considered metric corrections $h_{\alpha\beta}$ to proper distance D_p satisfying the vacuum perturbation equation $\nabla^2 h_{\alpha\beta} = 0$ in the flat space of FLRW matter-dominated cosmology, i.e., $\frac{d^2}{dD_p^2} h_{ii} = 0$ where D_p is proper distance per the FLRW metric [13,14]. We then corrected proper distance according to $D_p \to \sqrt{1 + h_{ii}}\, D_p = \sqrt{1 + \frac{D_p}{A}}\, D_p$ (A is an arbitrary constant used as a fitting parameter and was found to be about 8 Gcy). Here we adopt this approach for modeling missing mass phenomena. We will assume $dM_p = \sqrt{1+h} dM$ in analogy with our correction of proper distance above. Further, $\nabla^2 h = 0$ with spherical symmetry assumed for galactic rotation curves, X-ray cluster mass profiles, and the baryon-photon perturbations in pre-recombination FLRW cosmology. The important difference between our treatment of dark energy and missing mass is that while h satisfies the perturbation equation, it does not have to be the case that $h \ll 1$. But, as explained above, it is the case that even the enhanced proper mass constitutes a small perturbative correction to a background metric, so the motivation for this GR ansatz isn't totally ill-founded. Accordingly, the GR ansatz is based on $h = \frac{A}{r} + B$ which is used to obtain proper mass from dynamic mass per

$$dM_p = \sqrt{1+h}\, dM = \sqrt{\frac{A}{r} + B}\, dM \tag{5}$$

As with dark energy, the arbitrary constants A and B are used as fitting parameters. The best fit values of A and B show interesting trends across and within the three data sets [15].

3. Conclusion

Overall, the GR ansatz Eq(5) fits of THINGS data (average mean square error MSE = 101 (km/s)^2) compare well with MOND (d unconstrained average MSE = 51.7 (km/s)^2 and d constrained average MSE = 67.1 (km/s)^2), Burkett DM halo (average MSE = 119 (km/s)^2), and Navarro-Frenk-White (NFW) DM halo (average MSE = 149 (km/s)^2) (Figure(2)). As with galactic RC's, the GR ansatz fits of X-ray cluster mass profiles (ROSAT/ASCA data, average MSE = 0.00535 from $(\Delta \mathrm{Log}(M))^2$) compare well with metric skew-tensor gravity (MSTG) (average MSE = 0.0236) and core-modified NFW DM (average MSE = 0.00975) (Figure(3)). Finally, the GR ansatz fit of the Planck 2015 CMB angular power spectrum data (root-mean-square error RMSE = 225 $(\mu K)^2$) compares well with both ΛCDM and scalar-tensor-vector gravity (STVG) (Figure(4), both have RMSE = 240 $(\mu K)^2$). Twelve galactic RC's, eleven X-ray cluster mass profiles and the first three peaks of the CMB angular power spectrum do not provide enough data fits to draw any strong conclusions per se, but the general trends and results noted above are consistent with well-established research. Therefore, GR contextual mass arising from combined GR spacetimes modeling complex/realistic astrophysical matter distributions might provide a better model of astrophysical missing mass phenomena than Newtonian intrinsic mass.

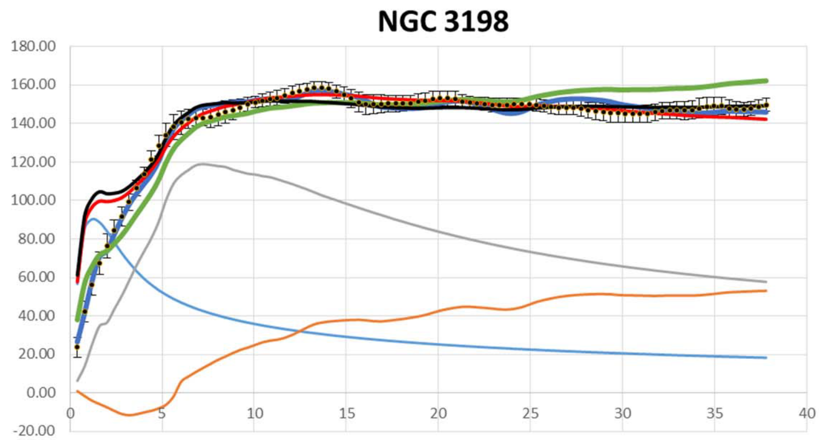

Figure 2. Graph of our GR ansatz fit (thick blue, MSE = 10.1), MOND fit (thick green, MSE = 18.8), Burkett DM halo fit (red, MSE = 72.9), and NFW DM halo fit (black, MSE = 101) of NGC 3198 THINGS galactic RC data (black dots with error bars). Disk contribution is grey, gas contribution is burnt orange, and bulge contribution is light blue. Vertical axis is rotation velocity in km/s, horizontal axis is orbital radius in kpc, and mean square error (MSE) is in $(\mathrm{km/s})^2$. Where the fits are crowded they all conform nicely to the data, so aberrant fitting regions are visible. Note: The MOND fit is with d unconstrained. Complete results can be found in [15].

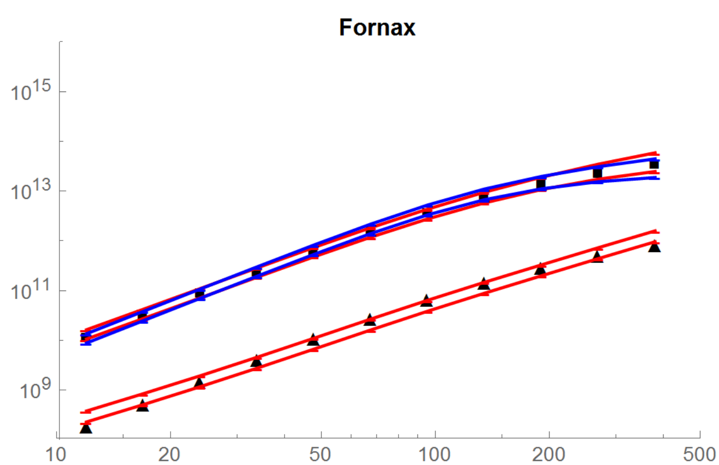

Figure 3. A log-log plot of our GR ansatz fit, MSTG fit, and core-modified NFW DM fit of the Fornax X-ray cluster mass profile (compiled from ROSAT and ASCA data). Vertical scale is in solar masses and horizontal scale is in kpc. Our GR ansatz (upper red lines, MSE = 0.00126) is increasing the gas (dynamic) mass (triangles) to fit the proper mass (squares). MSTG (lower red lines, MSE = 0.0128) is decreasing the proper mass to fit the gas (dynamic) mass. Core-modified NFW DM (upper blue lines, MSE = 0.00128) is adding matter to increase the gas (dynamic) mass to fit the proper mass. The sizes of the objects are approximately equal to their errors. Mean square error (MSE) is $(\Delta \mathrm{Log}(M))^2$. Line separation in the pair of lines (connecting fit points) corresponds to error. Complete results can be found in [15].

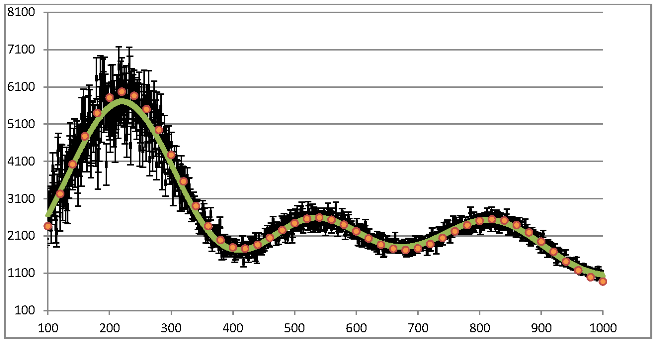

Figure 4. This is a plot of D_ℓ in $(\mu K)^2$ versus ℓ in the range $100 \leq \ell \leq 1000$ for the Planck 2015 CMB data [16] (black error bars), the Planck consortium's best ΛCDM fit [17] (solid green line), and our GR ansatz fit (Hu & Sugiyama [18,19] (HuS) standard cold dark matter (sCDM) fit) (orange dots). Our GR ansatz and STVG trivially reproduce the HuS fit without DM. Since we do not have Λ, our best fit to these data would equal the HuS sCDM best fit. The root-mean-square error (RMSE) for the HuS sCDM fit points shown is 225 $(\mu K)^2$. STVG can also trivially replace DM in ΛCDM and STVG keeps Λ, so the STVG best fit to these data would equal the ΛCDM best fit. The RMSE for the ΛCDM fit shown corresponding to the HuS fit points shown is 240 $(\mu K)^2$, although this fit is for all ℓ in the range $30 \leq \ell \leq 2508$. A derivation of these results can be found in [15].

References

[1] Brownstein, J.R.: Modified Gravity and the phantom of dark matter. PhD Thesis (2009) http://arxiv.org/abs/0908.0040

[2] Magalhaes, N.S., and Cooperstock, F.I.: Galactic mapping with general relativity and the observed rotation curves. (2015) http://arxiv.org/abs/1508.07491

[3] Carrick, J.D., and Cooperstock, F.I.: General relativistic dynamics applied to the rotation curves of galaxies. Astrophysics and Space Science 337, 321-329 (2012).

[4] Cooperstock, F.I., and Tieu, S.: Galactic dynamics via general relativity – A compilation and new results. International Journal of Modern Physics A 13, 2293-2325 (2007).

[5] Cooperstock, F.I.: The power of weak-field GR gravity. International Journal of Modern Physics D 25(12), 1644017 (2016) http://www.worldscientific.com/doi/pdf/10.1142/S021827181644017X

[6] Wald, R.: General Relativity. University of Chicago Press, Chicago (1984), p 126.

[7] Stuckey, W.M.: The observable universe inside a black hole. American Journal of Physics 62, 788-795 (1994).

[8] Misner, C.W., Thorne, K.S., and Wheeler, J.A.: Gravitation. W.H. Freeman, San Francisco (1973), Chapter 42, pp 851-853.

[9] Weinberg, S.: Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity. John Wiley & Sons, New York (1972), Chapter 3, pp 68-69.

[10] Stuckey, W.M., McDevitt, T., Sten, A.K., and Silberstein, M.: End of a Dark Age? International Journal of Modern Physics D 25(12) 1644004 (2016) https://arxiv.org/abs/1605.09229

[11] Moffat, J.W., and Rahvar, S.: The MOG weak field approximation and observational test of galaxy rotation curves. Monthly Notices of the Royal Astronomical Society 436, 1439-1451 (2013) https://arxiv.org/abs/1306.6383

[12] Moffat, J.W., and Rahvar, S.: The MOG Weak Field approximation II. Observational test of Chandra X-ray Clusters. Monthly Notices of the Royal Astronomical Society 441, 3724-3732 (2014) https://arxiv.org/abs/1309.5077

[13] Stuckey, W.M., McDevitt, T., and Silberstein, M.: Modified Regge Calculus as an Explanation of Dark Energy. Classical and Quantum Gravity 29, 055015 (2012a) http://arxiv.org/abs/1110.3973

[14] Stuckey, W.M., McDevitt, T., and Silberstein, M.: Explaining the Supernova Data without Accelerating Expansion. International Journal of Modern Physics D 21(11), 1242021 (2012b).

[15] Stuckey, W.M., McDevitt, T., Sten, A.K., and Silberstein, M.: The Missing Mass Problem as a Manifestation of GR Contextuality. https://arxiv.org/abs/1509.09288 (v5 24 Oct 2017).

[16] The file COM PowerSpect CMB R2.02.fits at http://irsa.ipac.caltech.edu/data/Planck/release 2/ancillary-data/HFI Products.html

[17] The file COM PowerSpect CMB-base-plikHM-TT-lowTEB-minimum-theory R2.02.txt at http://pla.esac.esa.int/pla/

[18] Hu, W., and Sugiyama, N.: Anisotropies in the Cosmic Microwave Background: An Analytic Approach. Astrophysical Journal 444(2), Part 1, 489-506 (1995) https://arxiv.org/abs/astro-ph/9407093

[19] Hu, W., and Sugiyama, N.: Small Scale Cosmological Perturbations: An Analytic Approach. Astrophysical Journal 471, 542-570 (1996).