This is not a very famous effect but it is really amazing. Consider a pendulum which consists of a massless rod of length $\boldsymbol{r}$ and a point of mass $\boldsymbol{m}$; the system is in the standard gravitational field $\boldsymbol{g}$. So the point $\boldsymbol{m}$ moves on the sphere of radius $\boldsymbol{r}$.
It is clear, the equilibrium when the point rests in the North Pole of the sphere is unstable.

Introduce a Cartesian inertial frame $OXYZ$ with origin in the point of suspension and the axis OZ is vertical such that $\boldsymbol{g} = -g\boldsymbol{e}_z$.
Now let us switch on a Lorentz force $\boldsymbol{F} = \boldsymbol{B} \times \boldsymbol{v}$ which acts on $\boldsymbol{m}$. The vector $\boldsymbol{B} = B\boldsymbol{e}_z$ is constant.

Theorem. Assume that B is sufficiently big:
$\frac{B^2}{8m} > \frac{mg}{2r}$
then the North Pole equilibrium is stable.

FIGURE 1. ...

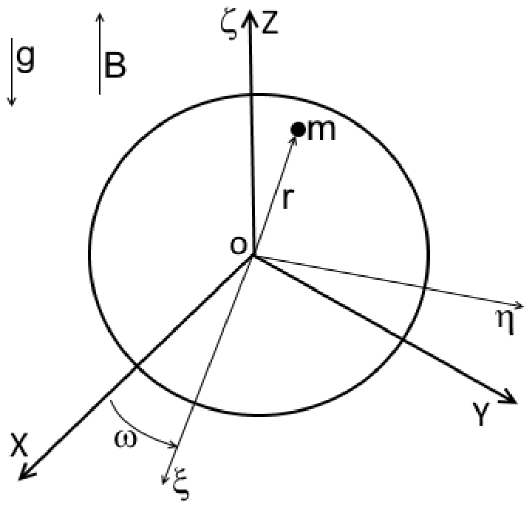

FIGURE 2. ...

The equation of particle's motion is

$$m\boldsymbol{a} = \boldsymbol{N} + m\boldsymbol{g} + \boldsymbol{B} \times \boldsymbol{v}, \qquad (*)$$

here $\boldsymbol{N} = N\boldsymbol{r}/r$ – is a reaction of the rod.

Introduce a moving frame $O\xi\eta\zeta$, such that axis $O\zeta$ coincides with Oz. The frame $O\xi\eta\zeta$ rotates about the axis $O\zeta$ with angular velocity $\boldsymbol{\omega} = \omega\boldsymbol{e}_z$. The constant ω to be determined.

Let $\boldsymbol{a}_e, \boldsymbol{v}_e$ stand for the acceleration and the velocity of the moving frame; $\boldsymbol{a}_c$ is the Coriolis acceleration; $\boldsymbol{v}_r, \boldsymbol{a}_r$ are the velocity and acceleration of the particle relatively the frame $O\xi\eta\zeta$. We have

$$\boldsymbol{a} = \boldsymbol{a}_r + \boldsymbol{a}_c + \boldsymbol{a}_e, \quad \boldsymbol{v} = \boldsymbol{v}_r + \boldsymbol{v}_e,$$

and

$$\boldsymbol{a}_e = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}), \quad \boldsymbol{a}_c = 2\boldsymbol{\omega} \times \boldsymbol{v}_r, \quad \boldsymbol{v}_e = \boldsymbol{\omega} \times \boldsymbol{r}.$$

Substitute these to the equation of motion

$$m\boldsymbol{a}_r = \boldsymbol{N} + m\boldsymbol{g} + \boldsymbol{B} \times (\boldsymbol{v}_r + \boldsymbol{\omega} \times \boldsymbol{r}) - m\big(\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times \boldsymbol{v}_r\big).$$

To get rid from the gyroscopic terms in the last formula choose

$$\boldsymbol{\omega} = \frac{1}{2m}\boldsymbol{B}.$$

It follows that

$$m\boldsymbol{a}_r = \boldsymbol{N} + m\boldsymbol{g} + \frac{1}{4m}\boldsymbol{B} \times (\boldsymbol{B} \times \boldsymbol{r}).$$

The last equation possesses the energy integral

$$\frac{1}{2}m|\boldsymbol{v}_r|^2 + V(\boldsymbol{r}), \quad V(\boldsymbol{r}) = -m(\boldsymbol{g}, \boldsymbol{r}) + \frac{1}{8m}|\boldsymbol{B} \times \boldsymbol{r}|^2.$$

This is checked by direct calculation. In the vicinity of the North Pole we have $\boldsymbol{r} = \xi\boldsymbol{e}_\xi + \eta\boldsymbol{e}_\eta + \sqrt{r^2 - \xi^2 - \eta^2}\boldsymbol{e}_\zeta$, and

$$V = \Big(\frac{B^2}{8m} - \frac{mg}{2r}\Big)\rho^2 + o(\rho^2),$$

as $\rho^2 = \xi^2 + \eta^2 \to 0$. QED