Heaviside's Compressed Electromagnetic Wave

Heaviside was troubled by the lack of a physical description of electromagnetic waves. He gave six examples in his Electromagnetic Theory books; none were satisfactory. He was particularly troubled by the physical meaning of the momentum carried by an electromagnetic wave. In his Electromagnetic Theory Vol. III, pp.144-158, Heaviside provided an answer to that question: his moving compressible ether. Here's the beginning of that chapter, which describes his reasoning (written in 1902):

CHAPTER X.
WAVES IN THE ETHER

Matter, Electricity, Ether and the Pressure of Radiation

503. On the very borders and extreme limits of Maxwell's Theory of the ether and its electromagnetic functions we come to definite indications that a wider theory is wanted. Not a narrower one, involving retrogression, but a wider one. Gravitation is left out in the cold, for one thing, although I have shown (v. 1, p. 455) that the propagation of the gravitational influence may be done at finite speed, which may be that of light, by relations which imitate the circuital equations of electromagnetics, and that the old idea that the speed of gravitation must be an enormous multiple of the speed of light in order to avoid aberration is only moonshine. Then there is the unknown nature of the connecting link between ether and matter. This is Maxwell's electrification. To call it an electron does not help much. It is true that Maxwell's idea of fundamental atoms of electricity has received very important experimental support, which may lead to better knowledge not merely of electrification, but of the nature of matter itself; but at present there is little definitely certain about the actual of any one of the three unknowns – matter, electrification, ether.

Then there is the moving force on the ether under electromagnetic influence. Maxwell's theory, to my mind, *proves* the existence of the force, by sound dynamics, and I must consider any theory that ignores it to contain unsound dynamics. (This, of course, assumes that the Maxwellian equations are true. If the force be denied, the circuital equations must be altered, or the distribution of energy.) But as this force exists, what does it do, what motions result? It does not follow necessarily that any motion results. The medium cannot be compressed if it be incompressible, or distorted if rigid. So incompressible rigid ether would simply oppose the force due to the electromagnetic cause by its own natural reaction. The moving force will do nothing in the way of motion or work.

But I think a rigid incompressible ether is an exceedingly difficult idea. How move matter and electrification through it without elastic or frictional resistance if it is, by hypothesis, rigid and incompressible? I think a deformable ether is far more probable. By the consideration of electromagnetic effects in a deformable medium I was led to my dynamic theory of the forces and stresses in the ether, which should not be confounded with Maxwell's static theory, as people will find out if they take the trouble to read it (*El. Pa.* V. 2, p. 524). There is nothing to show the necessity or even probability of incompressibility. If necessary, I should rather prefer to go to the other extreme as a hypothesis, and consider that the ether does not oppose compression elastically. This would do away with the difficulty about the motion of matter through the ether. Besides, there is some theoretical evidence, which counts for what it is worth. The only mechanical imitation of the circuital laws known is that furnished by the ideal rotational ether, when done for infinitely small motions. Now I have shown (vol. 2, p. 503) that in this analogy the existence of finite compressibility is inadmissible, if electrification is to keep its identity and not be dissipated in wave fashion, and that it is by assumption of no elastic resistance to compression that we come to Maxwell's purely transverse waves, with persistence of electrification. Even an analogy counts for something. But what about radiation from the Sun and stars? This constitutes progressive plane waves, practically. If Maxwell's "pressure of radiation" exists, its space variation along the ray is the moving force, so it must move a compressible ether and compress it, and it would appear from the circuital laws taken in the form given by me for a deformable medium that there must be a reaction upon the radiation itself, distorting the light waves and altering the speed of propagation. Now it a conclusion from observation that the speed of light in ether is constant, to a very high degree of constancy. Any variation of speed with amplitude that is permissible must be very minute. By consideration of stellar distances, therefore, it is possible to obtain rough ideas of the velocity permissibly produced in a compressible ether so as not to cause more than a stated small fraction of a wavelength of displacement of the antinodes in a wave train by the changed and variable value of the speed of propagation. In large scale electromagnetics, on the other hand, there might be large motions of the ether allowable, since the speed of propagation is so great as to allow of a sensibly instantaneous readjustment. But in making these calculations concerning disturbed radiation, which were rather troublesome and involved quite speculative data concerning the density and compressibility, I was induced to ask myself, why have the speed of radiation variable at all? Why not make it an absolute constant? It will save all this bother caused by variable speed. The following investigation was the result.

I follow that introduction with the derivation in modern SI units.

How to have Constant Speed through Space of Plane Radiation Traversing a Moving Compressible Ether.

For an isolated plane wave in Heaviside's Moving Compressible Ether there are two absolute constants: the speed of light and the characteristic impedance of free space. These are required to prevent the wave from becoming distorted over time.

Let c be the constant speed of radiation in plane waves, v the speed of propagation of electromagnetic waves through the medium, q the medium's speed along the x-axis, and z_0 the characteristic impedance of free space,

$$c = v + q = const \qquad (1)$$

$$z_0 = \sqrt{\frac{\mu}{\varepsilon}} = const \qquad (2)$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (3)$$

$$v\varepsilon = \frac{1}{z_0} = const \qquad (4)$$

$$v\mu = z_0 = const \qquad (5)$$

The equation of continuity of density in the medium (m) is,

$$-(mq)' = \dot{m} \tag{6}$$

where the prime represents the partial derivative with respect to x and the dot represents the partial derivative with respect to time. The equation of continuity of momentum in the wave is,

$$-c(mq)' = \frac{\partial}{\partial t}(mq) \tag{7}$$

Equations (6) and (7) result in a new constant, *mv*, which shows that the speed of propagation of the electromagnetic wave in the medium is inversely proportional to the density of the medium.

Newton's Second Law for a particular piece of the medium is,

$$F = m\frac{dq}{dt} = m(\dot{q} + qq') \tag{8}$$

which is the same as,

$$\boxed{F = -mvq'} \tag{9}$$

because $cq' = -\dot{q}$, and $v + q = c$.

Now it's time to bring in the electromagnetic force on the medium. The momentum in the wave is,

$$\vec{M} = \vec{D} \times \vec{B} = DB = \frac{B^2}{\mu v} = mq \tag{10}$$

and the force on the medium is the rate of change of electromagnetic energy along the wave,

$$\boxed{\vec{F} = -(\mu H^2)' = -\left(\frac{B^2}{\mu}\right)' = -(mvq)' = -mvq'} \tag{11}$$

Note that (9) and (11) are identical. One is based on purely medium dynamics and the other on electromagnetic considerations. Heaviside was happy with this result,

> "All conditions are fully satisfied. It is like magic. The result is, that radiation is transmitted by a compressible ether at absolutely constant speed through space if its [permeability] and permittivity vary as its density, and if the ether behaves as a substance subject to the moving force arising from the electromagnetic cause, whilst the electrical relations are the general ones introduced by me for a deformable medium."

Heaviside came back this topic again in 1909, Electromagnetic Theory Vol. III, pp. 422-432, this time starting with an infinitesimal volume element.