HS

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Equations

The self-consistent gap equation to be solved is given by:

$$\Delta_{s,d} = \frac{1}{N}\sum_k \left(v_{k1}u_{k3} + v_{k2}u_{k4}\right)\left(\cos(k_x) \pm \cos(k_y)\right)$$

Here, u_{k1} and v_{k2} are given by:

$$u_{k1}^2 = v_{k2}^2 = \frac{1}{2}\left[1 + \frac{\omega_\uparrow + \omega_\downarrow}{\sqrt{(\omega_\uparrow + \omega_\downarrow)^2 + 4\nu^2}}\right]$$

And u_{k2} and v_{k1} are given by:

$$u_{k2}^2 = v_{k1}^2 = \frac{1}{2}\left[1 - \frac{\omega_\uparrow + \omega_\downarrow}{\sqrt{(\omega_\uparrow + \omega_\downarrow)^2 + 4\nu^2}}\right]$$

The coefficients u_{k3}, u_{k4}, v_{k3}, and v_{k4} are given by:

$$u_{k3}^2 = v_{k4}^2 = \frac{1}{2}\left[1 + \frac{\omega_\uparrow + \omega_\downarrow}{\sqrt{(\omega_\uparrow + \omega_\downarrow)^2 + 4\nu^2}}\right]$$

$$u_{k4}^2 = v_{k3}^2 = \frac{1}{2}\left[1 - \frac{\omega_\uparrow + \omega_\downarrow}{\sqrt{(\omega_\uparrow + \omega_\downarrow)^2 + 4\nu^2}}\right]$$

where:

$$\omega_\sigma = h_{1\sigma}(k)(\alpha_{k\sigma}^2 - \beta_{k\sigma}^2) - 2h_{2\sigma}(k)\alpha_{k\sigma}\beta_{k\sigma}$$

$$\nu = -h_3(k)(\alpha_{k\uparrow}\beta_{k\downarrow} + \alpha_{k\downarrow}\beta_{k\uparrow})$$

where,

$$h_{1\sigma}(k) = \frac{U-\Delta}{2}\left(\frac{1+\delta-\sigma m_s}{2}\right) - \frac{t^2}{\Delta}\left[4(1-2\delta) + g_{1\sigma}(2\chi_{BB\bar{\sigma}} + 4\chi_{BB\bar{x}y\bar{\sigma}}) + g_{1\sigma}\frac{1-\delta+\sigma m_s}{2}\gamma_k\right] - \frac{2t^2}{U+\Delta}g_s\sigma m_s + \frac{2t^2}{U+\Delta}(1-\delta)$$

$$h_{2\sigma}(k) = \left[-tg_{1\sigma} - \frac{t^2}{\Delta}\left(-2\chi_{AB\sigma} + 6g_2\chi_{AB\bar{\sigma}}\right) - \frac{t^2}{U+\Delta}\left[g_s\left(\frac{1}{2}\chi_{AB\sigma} + \chi_{AB\bar{\sigma}}\right) + \frac{1}{2}\chi_{AB\sigma}\right]\right]\gamma_k$$

$$h_3(k) = \left[\frac{4t^2}{\Delta}(1-g_2) + \frac{4t^2}{U+\Delta}\left(\frac{3g_s}{4} - \frac{1}{4}\right) - \frac{2t^2}{\Delta}(g_{t\downarrow} + g_{t\uparrow})\right]\frac{\Delta_{AB}}{2}[\cos(k_x) - \cos(k_y)]$$

Here,

$$\gamma_k = 2[\cos(k_x) + \cos(k_y)]$$

$$\gamma'_k = 2[\cos(2k_x) + \cos(2k_y)] + 4[\cos(k_x + k_y) + \cos(k_x - k_y)]$$

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This results in,

$$2\alpha_{k\sigma}^2 = [1 - h_{1\sigma}(k)/E_\sigma(k)]$$

$$2\beta_{k\sigma}^2 = [1 + h_{1\sigma}(k)/E_\sigma(k)]$$

with,

$$E_\sigma(k) = \sqrt{h_{1\sigma}(k)^2 + h_{2\sigma}(k)^2}$$

At half-filling, the magnetizations on the A and B sub-lattices are equal and opposite to each other owing to the particle-hole symmetry. Hence, $m_s = (m_A - m_B)/2 = m_A$. Self-consistent equations for the various mean-field order parameters are

$$m_s = \frac{1}{N}\sum_k (\alpha_{k\uparrow}^2 - \alpha_{k\downarrow}^2),$$

$$\delta = \frac{1}{2N}\sum_{k\sigma} (\alpha_{k\sigma}^2 - \beta_{k\sigma}^2),$$

$$\chi_{AB\sigma} = -\frac{1}{4N}\sum_k \gamma_k \alpha_{k\sigma}\beta_{k\sigma},$$

$$\chi_{BB\sigma} = \frac{1}{N}\sum_k [\cos(2k_x) + \cos(2k_y)]\beta_{k\sigma}^2,$$

$$\chi_{BBxy\sigma} = \frac{1}{N}\sum_k 2\beta_{k\sigma}^2 \cos(k_x)\cos(k_y).$$

Also,

$$g_{t\sigma} = \frac{2\delta}{1 + \delta + \sigma m_s}$$

$$g_s = \frac{4}{(1+\delta)^2 - m_s^2}$$

$$g_1 = 1$$

$$g_2 = \frac{4\delta}{(1+\delta)^2 - m_s^2}$$

Here σ can be either $\uparrow$ or $\downarrow$; depending on the spin up or down respectively