Estimate the value of the mass function for this system. The period of the orbit is .423 days.

2. *The Roche Lobe*

   (a) Consider two point masses M_1 and M_2 held at fixed positions in space a distance d apart. Sketch contour lines of constant total Newtonian gravitational potential in a plane through the axis connecting the masses. Find the position between the stars at which the Newtonian gravitational force on a test particle vanishes.

   (b) Suppose the star with mass M_1 is surrounded by a fluid envelope whose mass contributes negligibly to the gravitational potential. Explain why the boundary of the envelope must lie on an equipotential. Sketch the shape of that boundary when material from the envelope is just about to flow onto the second mass. That is the *Roche lobe*. Compare with Figure 13.1.

   Comment: In this problem the masses were imagined to be fixed in space. In a model of a binary star system they would be rotating around one another. For a harder problem work out the shape of the Roche lobe taking proper account of this rotation.

3. The picture of the radio source Cygnus A in Figure 13.5 shows only one jet from the central source. Rotating black hole models of the source suggest there could be *two* jets emerging in opposite directions along the rotation axis. What famous effect of special relativity could contribute to an explanation of why one jet is visible and the other not? Assuming the intensities differ by a factor of 100, and that visible jet makes an angle of 45° with respect to the line of sight, what can you say about the velocity of the sources of visible radiation in the jets?

4. Figure 13.4 shows the orbits of stars around the $3 \times 10^6 M_\odot$ black hole at the center of our galaxy approximately 9 kpc (kpc = kiloparsec) away. Calculate the predicted linear orbital velocities as a function of angular separation from the center assuming that the stars are in circular, Newtonian orbits whose plane is perpendicular to the line of sight. How do your results compare with the velocities that can be estimated from the angular positions that are shown over several years?

5. What is the mass of a black hole formed at the beginning of the universe that would explode by the Hawking process at the time the universe becomes transparent to radiation—approximately 400,000 yr after the big bang?

6. [E] *Estimate* how long an electron-positron pair created in a vacuum fluctuation can last, assuming that the fluctuation can violate energy conservation for a time Δt consistent with the energy-time uncertainty principle $\Delta E \Delta t > \hbar$.

7. [E] *Estimate* the distance at which the energy received at Earth from an exploding primordial black hole in the last one second of its life would be comparable to that received from a nearby star in the same period. (For definiteness take the star to have the luminosity of the Sun and be 10 pc away.)

15. [C] *Temperature of a Rotating Black Hole*

(a) An axisymmetric body is spinning about its symmetry axis with angular velocity Ω and angular momentum J along the axis. Show that, in Newtonian mechanics, the work required to increase the angular momentum by a small amount ΔJ is $\Omega \Delta J$.

(b) Reorganize (15.36) for the change in area A of a rotating black hole given changes in its mass and angular momentum into a form like the first law of thermodynamics, assuming that the entropy of the black hole is $k_B A/4\hbar$, as in the Schwarzschild case [cf. (13.18)]. Find the Hawking temperature of a Kerr black hole.

(c) Show that the temperature of an extreme ($a = M$) black hole is zero and explain this fact from properties of the Kerr geometry.

16. [B, E] An active galactic nucleus with a luminosity of 10^{46} ergs is powered by the rotational energy of an extreme rotating black hole as described in Box 15.1 on p. 326. *Estimate* how long the active galactic nucleus can radiate in this way. Compare your answer to the present age of the universe, approximately 15 billion years.

17. [B, P, S] Show that (b) in Box 15.1 on p. 326 gives the voltage developed across any axisymmetric conductor rotating around its symmetry axis.

18. [B, E, P] Consider a rotating black hole with $M \sim 10^9 M_\odot$, $\Omega_H M \sim 1$ immersed in a magnetic field $B \sim 10^4$ gauss as described in Box 15.1. *Estimate* how far an electron in the vicinity of the black hole has to move in the electric field there before it acquires enough energy to make a further electron-positron pair in a collision with a similar electron or positron.

2. Suppose that the scale factor describing the expansion of the universe is

$$a(t) = (t/t_*)^{1/2},$$

where t_* is a constant and t is the proper time from the singularity. Suppose that the present age of the universe is 14 Gy.

(a) What would be the value (in yr^{-1}) of the Hubble constant observed today?

(b) At what age in years would the temperature of the microwave background be 3000 K?

3. Consider a flat FRW model whose metric is given by (18.1). Show that, if a particle is shot from the origin at time t_* with a speed V_* as measured by a comoving observer (constant x, y, z), then asymptotically it comes to rest with respect to a comoving frame. Express the comoving coordinate radius at which it comes to rest as an integral over $a(t)$.

4. [S] Suppose the present value of the Hubble constant is 72 (km/s)/Mpc and that the universe is at critical density. A photon is emitted from our galaxy now. What is the redshift of this photon when it is received in another galaxy 10 billion years in the future, assuming it continues to be matter dominated?

5. [S] The cosmic background radiation has been propagating to us since the universe became transparent at a temperature of approximately 3000 K. Its temperature today is 2.73 K. What is the redshift z of the radiation?

6. [S] A type Ia supernova has a redshift of $z = 1.1$. The observed brightness rises and falls on a timescale of two months. (More precisely let's say the difference in times between when the supernova is at half peak brightness is two months.) What is the timescale for the rise and fall in the supernova's rest frame as would be seen by a hypothetical observer close to the supernova and at rest with respect to it?

7. Consider a galaxy whose light we see today at time t_0 that was emitted at time t_e. Show that the present proper distance to the galaxy (along a curve of constant t_0) is

$$d = a(t_0) \int_{t_e}^{t_0} dt/a(t).$$

8. In Section 9.2 the redshift of a photon in the Schwarzschild geometry was derived using the conservation law arising from time-translation symmetry. Show that the cosmological red shift (18.10) can be derived from the *space* translation symmetry of the metric (18.1) in a similar way.

9. [E] *Estimate* in centimeters the size of the universe visible today at the time the CMB radiation last interacted with matter at a temperature of approximately 3000 K.

10. [E, C] As the universe expands, the horizon grows. *Estimate* the time it has to grow for one new galaxy to come within the horizon, assuming the universe was matter dominated over the whole of its history.

11. (a) Equation (18.69) gives the scale factor as a function of time for closed, matter-dominated FRW models. Show that, if the parameter η that occurs there is used as a time coordinate, the FRW metric takes the form

$$ds^2 = a^2(\eta)[-d\eta^2 + d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)].$$