

2.1 Consider an array of  $N$  localised spin- $\frac{1}{2}$  paramagnetic atoms. In the presence of a magnetic field,  $B$ , the two degenerate spin states split by  $\pm\mu_{\text{B}}B$ , where  $\mu_{\text{B}}$  is the Bohr magneton.

(i) Derive the single particle partition function for the system.

(ii) Show that the heat capacity  $C$  can be written as

$$C = \frac{dU}{dT} = Nk_{\text{B}} \left( \frac{\theta}{T} \right)^2 \frac{e^{\theta/T}}{(e^{\theta/T} + 1)^2}, \quad (5)$$

and find the value of the constant  $\theta$ . Show that  $C$  has a peak at a temperature  $T_{\text{peak}} = A\mu_{\text{B}}B/k_{\text{B}}$  where  $A$  is a numerical constant. Determine  $A$ .

(iii) Given that the largest static magnetic field that can easily be produced in the laboratory is of order 2 Tesla, estimate the temperature at which the magnetic heat capacity of such a system will be important.

(iv) Show that in the limits of high and low temperatures the heat capacity is proportional to  $1/T^2$  as  $T \rightarrow \infty$  and proportional to  $e^{-\theta/T}/T^2$  as  $T \rightarrow 0$ .