$$\mathrm{E}[\exp(Vt)] = \exp((\mu + \frac{\sigma^2\theta}{2})t + \frac{t^2}{2}(\frac{\sigma^2}{n} + \frac{\sigma^4\theta^2}{2(n-1)}).$$

$$\mathrm{E}[\exp V] = \exp(\mu + \frac{\sigma^2\theta}{2} + \frac{\sigma^2}{2} + \frac{\sigma^4\theta^2}{4(n-1)})$$

where $E[\exp V] = E[\hat{\alpha}_2]$.

When θ=1, α is consistent. I'm meant to find a value of θ so as to reduce the bias of the estimator in comparison to θ=1. Any ideas?

I use the CLT to assume that $V = \bar{Y} + S_Y^2$ is normally distributed and thus have the mgf of V which is the equation above when t=1.