We start with the Hermite approximation. The Hermite curve is a degree-3 polynomial in *t* with four coefficients that depend on the two points and two tangents.

$P(t) = at^3 + bt^2 + ct + d = (t^3, t^2, t, 1)(a, b, c, d)^T = T(t)A$

This is the algebraic representation of the curve, in which the four coefficients are still unknown. Once these coefficients are expressed in terms of the known quantities, which are geometric, the curve will be expressed geometrically.

The tangent vector to a curve P(*t*) is the derivative dP(t)/dt, which is denoted by Pt(t). The tangent vector of a curve is therefore

$Pt(t) = 3at^2 + 2bt + c$

Bringing polynomial and coefficients into correlation yields following expressions

$a0^3 + b0^2 + c0 + d = P1$
$a1^3 + b1^2 + c1 + d = P2$
$3a0^2 + 2b0 + c = Pt1$
$3a1^2 + 2b1 + c = Pt2$

which after solving have this form

$a = 2P1 - 2P2 + Pt1 + Pt2$
$b = -3P1 + 3P2 - 2Pt1 - Pt2$
$c = Pt1$
$d = P1$

Substituting

$P(t) = (2P1 - 2P2 + Pt1 + Pt2)t^3 + (-3P1 + 3P2 - 2Pt1 - Pt2)t^2 + Pt1t + P1$

and rearranging

$P(t) = (2t^3 - 3t^2 + 1)P1 + (-2t^3 + 3t^2)P2 + (t^3 - 2t^2 + t)Pt1 + (t^3 - t^2)Pt2$

After substitution

$F1(t) = (2t^3 - 3t^2 + 1)$
$F2(t) = (-2t^3 + 3t^2) = 1 - F1(t)$
$F3(t) = (t^3 - 2t^2 + t)$
$F4(t) = (t^3 - t^2)$

the curve can be written in the form

$$P(t) = F(t)B = T(t)HB = \begin{pmatrix} t^3 & t^2 & t^1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P1 \\ P2 \\ Pt1 \\ Pt2 \end{pmatrix}$$

which represents uniform Hermite curve.

As the length over t of underlying Limacon segment is NOT $0<=t<=1$, the approximation with $0<=t<=1$ will not yield the smooth curve. So, instead of letting $0<=t<=1$, we let it $0<=t<=\Delta$, whereby Δ is any positive real number. This non uniform Hermite is known under Hermite segment with tension.

The derivation of this case is similar to the uniform case.

$a0^3 + b0^2 + c0 + d = P1$
$a\Delta^3 + b\Delta^2 + c\Delta + d = P2$
$3a0^2 + 2b0 + c = Pt1$
$3a\Delta^2 + 2b\Delta + c = Pt2$

with solutions

$a = 2(P1 - P2)/\Delta^3 + (Pt1 + Pt2) / \Delta^2$
$b = 3(P2 - 3P1)/\Delta^2 - 2Pt1/\Delta - Pt2/\Delta$
$c = Pt1$
$d = P1$

which is actually equal to

$$Pnu(t) = \begin{pmatrix} t^3 & t^2 & t^1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} P1 \\ P2 \\ \Delta Pt1 \\ \Delta Pt2 \end{pmatrix}$$

Through introduction of correction factor Δ we can let t vary from 0<=t<=1 again.

Using expression Pnu(t) for any arbitrary point on the curve we can determine Δ. As the underlying Limacon expression in parametric form is known (X(t) and Y(t)), for t=0.5 for instance) we can write equation.

$P(th) = (2P1 - 2P2 + \Delta Pt1 + \Delta Pt2)t^3 + (-3P1 + 3P2 - 2\Delta Pt1 - \Delta Pt2)t^2 + \Delta Pt1t + P1$

for arbitrary segment starting with t1 ending with t2, we have th = t1 + (t2 – t1) /2

Subdivided on X and Y coordinates we get two equations

$$\begin{aligned} X(th) = {} & (2X(t1) - 2X(t2) + \Delta xX'(t1) + \Delta xX'(t2))t^3 + \\ & (-3X(t1) + 3X(t2) - 2\Delta xX'(t1) - \Delta xX'(t2))t^2 + \\ & \Delta xX'(t1)t + \\ & P1 \end{aligned}$$

$$\begin{aligned} Y(th) = {} & (2Y(t1) - 2Y(t2) + \Delta yY'(t1) + \Delta yY'(t2))t^3 + \\ & (-3Y(t1) + 3Y(t2) - 2\Delta yY'(t1) - \Delta yY'(t2))t^2 + \\ & \Delta yY'(t1)t + \\ & P1 \end{aligned}$$

with unique solutions

$\Delta x = 4 (2X(th) - X(t1) - X(t2)) / (X'(t1) - X'(t2))$
$\Delta y = 4 (2Y(th) - Y(t1) - Y(t2)) / (Y'(t1) - Y'(t2))$

which can be applied back to the Pnu(t) statement and have the Hermite curve pass through the specified point t=0.5 of the segment.

NOTE:
Parts of this description is based upon the book:
Curves and Surfaces for Computer Graphics, by David Salomon, Springer 2006