

Let's find the coordinate of B when A sends the beam:

$$x' = \frac{x + vt}{\sqrt{1 - v^2/c^2}} = \frac{0 + 0.5ct}{\sqrt{3}/2} = \frac{ct}{\sqrt{3}}$$

Now let's calculate the light reaches B:

$$x_{\text{light}} = x_B \Rightarrow c(T - t) = \frac{cT}{\sqrt{3}}$$

$$\therefore T = \frac{t\sqrt{3}}{\sqrt{3} - 1} \quad (T \text{ is the time in frame A, starting when B leaves A}).$$

Now let's calculate the coordinate of B at this time:

$$x' = \frac{c \times \frac{t\sqrt{3}}{\sqrt{3} - 1}}{\sqrt{3}} = \frac{ct}{\sqrt{3} - 1}$$

Now B delays  $2t$  seconds in its frame, so its time becomes  $T' = 2t$

$$2t = \frac{\Delta t}{\sqrt{3}/2} \Rightarrow \Delta t = \frac{4t}{\sqrt{3}} \quad (\text{time change is A reference frame})$$

Now let's find the coordinate of B at this time:

$$x' = \frac{c}{\sqrt{3}} \times \left( \frac{t\sqrt{3}}{\sqrt{3} - 1} + \frac{4t}{\sqrt{3}} \right)$$

So now B sends a light beam back to A, so it travels

$\Delta t$  seconds back to A:

$$c(\Delta t) = \frac{c}{\sqrt{3}} \times \left( \frac{t\sqrt{3}}{\sqrt{3} - 1} + \frac{4t}{\sqrt{3}} \right)$$

$$\therefore \Delta t = \frac{t}{\sqrt{3} - 1} + \frac{4t}{3}$$

So the total time:

$$T_{\text{total}} = \frac{t\sqrt{3}}{\sqrt{3} - 1} + \frac{4t}{\sqrt{3}} + \frac{t}{\sqrt{3} - 1} + \frac{4t}{3} =$$

$$= \frac{3\sqrt{3}t + 4\sqrt{3}(\sqrt{3} - 1)t + 3t + 4t(\sqrt{3} - 1)}{3(\sqrt{3} - 1)} =$$

$$= \frac{t}{3(\sqrt{3} - 1)} \cdot \frac{(3\sqrt{3} + 11)(\sqrt{3} + 1)}{(\sqrt{3} + 1)} = \frac{t(20 + 14\sqrt{3})}{6} =$$

$$= \frac{t(10 + 7\sqrt{3})}{3}$$