MATH 1336
Homework #2
Due Friday, October 20, 2017

Please show all work in order to receive credit.

1. This problem examines the series $\sum_{n=1}^{\infty} n\sin(\pi n)$. (1 point each)

   (a). Evaluate the improper integral $\int_1^{\infty} x\sin(\pi x)\,dx$.

   (b). Try writing out the first several terms of the series. Does it converge or diverge? Why?

   (c). Do your answers to parts (a) and (b) agree? Why does this not contradict the Integral Test?

2. Determine whether the following series converge or diverge. (3 points each)

   (a). $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ (b). $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ (c). $\displaystyle\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

3. (2 points) Let p_n denote the nth prime number, so $p_1 = 2$, $p_2 = 3$, $p_3 = 5$, $p_4 = 7$, $p_5 = 11$, etc. It is known that there is a positive constant C such that $p_n \le Cn\ln n$ for all n. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{p_n} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \cdots$ converges or diverges.

4. The nth *Harmonic number* is defined as

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} \cdots + \frac{1}{n}.$$

   Let $a_n = H_n - \ln n$. (2 points each)

   (a). Show that $a_n \ge 0$ for $n \ge 1$. [Hint: draw a picture to show that $H_n \ge \displaystyle\int_1^{n+1} \frac{1}{x}\,dx$.]

   (b). Show that $\{a_n\}$ is decreasing. [Hint: interpret $a_n - a_{n+1}$ as an area.]

   (c). Show that $\displaystyle\lim_{n\to\infty} a_n$ exists.