Homework 5
Due: November 20, 2014

1. For the given potential energies, sketch their graph, calculate the force they correspond to and identify the stable and unstable equilibrium points. (All the constants appearing below or positive constants).

   (a) $U = \frac{1}{2}kx^2$

   (b) $U = \lambda(x^2 - x_0^2)^2$

   (c) $U = \begin{cases} A\sin(kx) & \text{if } |x| \leq \frac{5\pi}{2k} \\ A\left(|x| - \frac{5\pi}{2k}\right)^2 & \text{otherwise} \end{cases}$

   (d) $U = -\frac{alpha}{r} + \frac{M^2}{2mr^2}$, where $r = \sqrt{x^2 + y^2 + z^2}$

2. A mass m is moving towards and inclined plane with an initial speed v_0. When it reaches the inclined plane, it feels a friction force with kinetic friction coefficient μ_k. How long will the mass travel before stopping? (Solve using energy relations, i.e. non-conservation of energy)

3. Consider two large planets with masses m_1 and m_2. They are separated by a distance R. A small mass m is placed at a distance d from the planet with mass m_1, along the line connecting the planets.

   (a) What should be the distance $d$such that the mass $m$does not feel any force?

   (b) Assume that the mass is slightly displaced by $x \ll d$ from its equilibrium point along the line connecting the planets. What is the force it will feel? Is the direction of the force towards or away from the equilibrium point?

   (c) Assume that the mass is slightly displaced by $x \ll d$ from its equilibrium point along a line perpendicular to the line connecting the planets. What is the force it would feel? Is the direction of the force towards or away from the equilibrium points?

   (*Hint:* If $x \ll 1$, $(1 + x)^n \simeq 1 + nx$).

4. Assume that you do not know the gravitational force that a spherical uniform mass distribution creates. Starting from the gravitation potential energy created by a point mass, calculate the potential energy

of a system consisting of a uniform spherical mass M and a point mass m. Assume that the sphere has radius R and the mass m is at a distance R from the centre of the mass M. Using the potential you have calculated, calculate the force gravitational force that mass m feels.