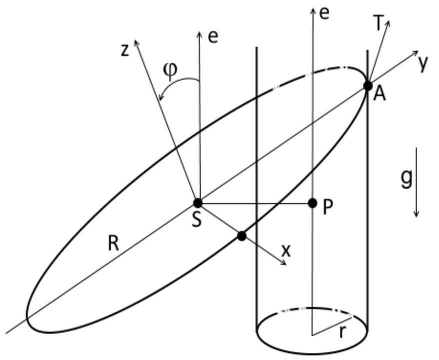

Рис. 1. ...

Let S be the centre of hoop and let φ be the angle between the horizontal plane and hoop's plane. The letter A stands for the contact point of the hoop and the pillar.

Introduce a moving Cartesian frame $Sxyz$. The axis Sx remains horizontal (we are looking for such a motion) and belongs to the hoop's plane; the axis Sy paths through the point

$$A, \quad \boldsymbol{SA} = R\boldsymbol{e}_y.$$

Correspondingly, the axis Sz is perpendicular to the plane of hoop.

Then $\boldsymbol{e} = \cos\varphi\boldsymbol{e}_z + \sin\varphi\boldsymbol{e}_y$ is the unit vertical vector, $\boldsymbol{g} = -g\boldsymbol{e}$. Let $\boldsymbol{T}$ stand for the reaction force of the pillar.

The equations of hoop's motion are

$$J_S\boldsymbol{\varepsilon} + [\boldsymbol{\omega}, J_S\boldsymbol{\omega}] = [\boldsymbol{SA}, \boldsymbol{T}], \quad m\boldsymbol{a}_S = m\boldsymbol{g} + \boldsymbol{T}.$$

Here m is the mass of hoop; $J_S = mR^2\mathrm{diag}\,(1/2, 1/2, 1)$ is the hoop's inertia tensor with respect to the point S; $\boldsymbol{\varepsilon}, \boldsymbol{\omega}$ are hoop's angular acceleration and angular velocity respectively; $\boldsymbol{a}_S$ is the acceleration of the hoop's centre.

From these two equation one obtains

$$J_S\boldsymbol{\varepsilon} + [\boldsymbol{\omega}, J_S\boldsymbol{\omega}] = m[\boldsymbol{SA}, \boldsymbol{a}_S - \boldsymbol{g}]. \qquad (*)$$

The angular velocity of the frame $Sxyz$ is expressed as follows $\boldsymbol{\omega}_e = \Omega\boldsymbol{e}$; and $\boldsymbol{\omega}_r = \nu\boldsymbol{e}_z$ is the angular velocity of the hoop in its motion relatively to the frame $Sxyz$.

We are looking for the motion such that the values ν, Ω, φ are constants.

Thus the absolute angular velocity of the hoop is given by the formula $\boldsymbol{\omega} = \boldsymbol{\omega}_e + \boldsymbol{\omega}_r$. The absolute angular acceleration is $\boldsymbol{\varepsilon} = [\boldsymbol{\omega}_e, \boldsymbol{\omega}_r] = \Omega\nu\sin\varphi\boldsymbol{e}_x$.

Introduce a horizontal vector

$$\boldsymbol{PS} = (R\cos\varphi - r)(-\cos\varphi\boldsymbol{e}_y + \sin\varphi\boldsymbol{e}_z), \quad R\cos\varphi - r > 0.$$

Then the velocity of the point S is $\boldsymbol{v}_S = [\boldsymbol{\omega}_e, \boldsymbol{PS}]$, and

$$\boldsymbol{a}_S = [\boldsymbol{\omega}_e, \frac{d}{dt}\boldsymbol{PS}] = [\boldsymbol{\omega}_e, [\boldsymbol{\omega}_e, \boldsymbol{PS}]].$$

Substituting all these formulas to (*), we yield

$$2R\Omega\nu\sin\varphi + \Omega^2\sin\varphi(3R\cos\varphi - 2r) - 2g\cos\varphi = 0. \qquad (**)$$

Non-slip condition $\boldsymbol{v}_S = [\boldsymbol{\omega}, \boldsymbol{AS}] = [\boldsymbol{\omega}_e, \boldsymbol{PS}]$ implies $\nu = -r\Omega/R$. Substitute this formula to (**) and find

$$\Omega^2 = \frac{2g\cos\varphi}{\sin\varphi(3R\cos\varphi - 4r)}.$$

Consequently, one has $3R\cos\varphi - 4r > 0$, particularly this gives $R > 4r/3$.

The normal component to pillar's surface of the vector $\boldsymbol{T}$ is

$$T_n = m\Omega^2(R\cos\varphi - r).$$

The tangent component is $T_\tau = mg$. So that we have $T_\tau < \mu T_n$, here μ is the friction coefficient.