

Simulation of the ice-cream ordering problem from Physics Forums. The five types of ice creams are  $\{1,2,3,4,5\}$ , of which we choose 3 at random (without replacement), and are interested in  $B$  = number of samples containing ice-cream '1'. There are four toppings  $\{1,2,3,4\}$ , of which we choose 2 and are interested in  $C$  = number of samples containing topping '1'. We are also interested in  $BC$  = number of samples containing both ice-cream '1' and topping '1'.

Below, we use the Maple package 'randcomb(n,k)' to pick a random subset of size  $k$  from  $\{1,2,\dots,n\}$ , then test if 1 is in the subset. We do that by naming the subset, then doing a Boolean evaluation (evalb()) to see if the statement "1 in the subset" is true or false. We then convert true/false to 1/0 for purposes of tallying up the totals. We do the same type of operations for choosing the toppings.

With each draw we leave  $B$  unchanged (if the 'flavor' subset does not contain '1') or increment it by 1 otherwise. Similarly for the toppings total  $C$ . Finally, we increment  $BC$  by 1 if and only if both  $B$  and  $C$  were incremented.

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> restart;
> with(Statistics) :
> with(combinat);
[Chi, bell, binomial, cartprod, character, choose, composition, conjpart, decodepart,      (1)
 encodepart, eulerian1, eulerian2, fibonacci, firstcomb, firstpart, firstperm, graycode,
 inttovec, lastcomb, lastpart, lastperm, multinomial, nextcomb, nextpart, nextperm,
 numbbcomb, numbbcomp, numbbpart, numbbperm, partition, permute, powerset, prevcomb,
 prevpart, prevperm, randcomb, randpart, randperm, rankcomb, rankperm, setpartition,
 stirling1, stirling2, subsets, unrankcomb, unrankperm, vectoint]
> randomize( ) : # <--- initialize the random number generator
> B := 0; C := 0; BC := 0; # <-- initial totals
                                     B := 0
                                     C := 0
                                     BC := 0                                     (2)
>
> N := 10000; # <-- number of samples
                                     N := 10000                                (3)
> for i from 1 to N do
  xb := 1 in randcomb(5, 3) :
  Ib := subs( {false = 0, true = 1}, evalb(xb) ) :
  xc := 1 in randcomb(4, 2) :
  Ic := subs( {false = 0, true = 1}, evalb(xc) ) :
  B := B + Ib : C := C + Ic : BC := BC + Ib·Ic :
end do:
> B, C, BC;
                                     6075, 4980, 3027                                (4)

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So, in this sample of 10,000 ice-cream cone orders:

(1) flavor '1' occurred  $B = 6075$  times (so the sample probability is 0.6075, quite close to the theoretical 60%);

- (2) the topping '1' occurred  $C = 4980$  times (so the sample probability is 0.4980, quite close to the theoretical 50%);
- (3) the flavor-topping combination '1 1' occurred  $BC = 3027$  times (so the sample probability is 0.3027, quite close to the theoretical 30%)

