I have proved That a vector field can be decomposed in

$$\bar{v}(r) = -\nabla\left\{\int_V \frac{\nabla'\cdot\bar{v}(r')}{4\pi(\bar{r}-\bar{r}')}\,dv' - \int_{\partial V}\frac{\bar{v}(r')\cdot\bar{n}}{4\pi(\bar{r}-\bar{r}')}\,da'\right\} \quad *$$

$$+\nabla\times\left\{\int_V \frac{\nabla'\times\bar{v}(\bar{r}')}{4\pi(\bar{r}-\bar{r}')}\,dv' + \int_{\partial V}\frac{\bar{v}(r')\times\bar{n}}{4\pi(\bar{r}-\bar{r}')}\,da'\right\}$$

Then I have foud different defifinitions of helmoTz-decomposition in bounded and unbounded domains, with 2 or 3 Terms

- in unbounded domains wher $\bar{v}(r)$ vanish at ∞ the Two surface integrals vanish and we have

$$\bar{v}(\bar{r}) = -\nabla D + \nabla\times\bar{R}$$

- in bounded domains I have found Two different forms of The Theorem That are valid with proper boundary conditions

(i) $\bar{v}(\bar{r}) = \nabla D + \bar{v}$

(ii) $\bar{v}(\bar{r}) = \nabla D + \nabla\times\bar{R} + \bar{h}$

$\downarrow$

is not clear To me the corrispondence of The Terms in These To forms of The decomposiTion with The expanded one *

I Think is

$$D = \int_V \frac{\nabla'\cdot\bar{v}(r')}{4\pi(r-r')}\,dv'$$

$$\bar{R} = \int_V \frac{\nabla'\times\bar{v}(r')}{4\pi(r-r')}\,dv'$$

$$\bar{v} = \nabla\times\left[\int_V \frac{\nabla'\times\bar{v}(r')}{4\pi(r-r')}\,dv' + \int_{\partial V}\frac{\bar{v}(r')\times\bar{n}}{4\pi(r-r')}\,da'\right] + \nabla\int_{\partial V}\frac{\bar{v}(r')\cdot\bar{n}}{4\pi(\bar{r}-\bar{r}')}\,da'$$

$$h = -\nabla\int_{\partial V}\frac{\bar{v}(r')\cdot\bar{n}}{4\pi(r-r')}\,da' + \nabla\times\int_{\partial V}\frac{\bar{v}(r')\times\bar{n}}{4\pi(r-r')}\,da'$$