3.2.5 If A is an $n \times n$ matrix, show that

$$\det(-\mathsf{A}) = (-1)^n \det \mathsf{A}.$$

3.2.6 (a) The matrix equation $\mathsf{A}^2 = 0$ does not imply $\mathsf{A} = 0$. Show that the most general 2×2 matrix whose square is zero may be written as

$$\begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix},$$

where a and b are real or complex numbers.

(b) If $\mathsf{C} = \mathsf{A} + \mathsf{B}$, in general

$$\det \mathsf{C} \neq \det \mathsf{A} + \det \mathsf{B}.$$

Construct a specific numerical example to illustrate this inequality.

3.2.7 Given the three matrices

$$\mathsf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \mathsf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathsf{C} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

find all possible products of A, B, and C, two at a time, including squares. Express your answers in terms of A, B, and C, and 1, the unit matrix. These three matrices, together with the unit matrix, form a representation of a mathematical group, the **vierergruppe** (see Chapter 4).

3.2.8 Given

$$\mathsf{K} = \begin{pmatrix} 0 & 0 & i \\ -i & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix},$$

show that

$$\mathsf{K}^n = \mathsf{KKK}\cdots (n \text{ factors}) = 1$$

(with the proper choice of n, $n \neq 0$).

3.2.9 Verify the **Jacobi identity**,

$$[\mathsf{A}, [\mathsf{B}, \mathsf{C}]] = [\mathsf{B}, [\mathsf{A}, \mathsf{C}]] - [\mathsf{C}, [\mathsf{A}, \mathsf{B}]].$$

This is useful in matrix descriptions of elementary particles (see Eq. (4.16)). As a mnemonic aid, the you might note that the Jacobi identity has the same form as the *BAC–CAB* rule of Section 1.5.

3.2.10 Show that the matrices

$$\mathsf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathsf{B} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathsf{C} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

satisfy the commutation relations

$$[\mathsf{A}, \mathsf{B}] = \mathsf{C}, \qquad [\mathsf{A}, \mathsf{C}] = 0, \qquad \text{and} \qquad [\mathsf{B}, \mathsf{C}] = 0.$$