$$|\psi\rangle = e^{-i\frac{\hat{H}}{\hbar}t}|\psi_0\rangle \qquad \hat{H} = \hbar\omega\hat{L}_y$$

eigenvalue

$$= e^{-i\omega\hat{L}_y t}|\psi_0\rangle \qquad f(\hat{A}) = \sum_i f(a_i)|a_i\rangle\langle a_i|$$

$$\hat{L}_y = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad e^{-i\omega\hat{L}_y t} = e^0|v_1\rangle\langle v_1| + e^{-i\omega\sqrt{2}t}|v_2\rangle\langle v_2| + e^{+i\omega\sqrt{2}t}|v_3\rangle\langle v_3|$$

$$= 1\cdot\frac{1}{2}\begin{pmatrix}1\\0\\1\end{pmatrix}\begin{pmatrix}1 & 0 & 1\end{pmatrix} + \frac{e^{-i\omega\sqrt{2}t}}{4}\begin{pmatrix}-1\\-i\sqrt{2}\\1\end{pmatrix}\begin{pmatrix}-1 & +i\sqrt{2} & 1\end{pmatrix} + \frac{e^{i\omega\sqrt{2}t}}{4}\begin{pmatrix}-1\\i\sqrt{2}\\1\end{pmatrix}\begin{pmatrix}-1 & -i\sqrt{2} & 1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0 & 1\\0 & 0 & 0\\1 & 0 & 1\end{pmatrix} + \frac{e^{-i\omega\sqrt{2}t}}{4}\begin{pmatrix}1 & -i\sqrt{2} & -1\\ i\sqrt{2} & 2 & -i\sqrt{2}\\ -1 & i\sqrt{2} & 1\end{pmatrix} + \frac{e^{i\omega\sqrt{2}t}}{4}\begin{pmatrix}1 & i\sqrt{2} & -1\\ -i\sqrt{2} & 2 & i\sqrt{2}\\ -1 & -i\sqrt{2} & 1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0 & 1\\0 & 0 & 0\\1 & 0 & 1\end{pmatrix} + \frac{1}{2}\begin{bmatrix} \frac{e^{-i\omega\sqrt{2}t}}{2} + \frac{e^{i\omega\sqrt{2}t}}{2} & i\sqrt{2}\left(-\frac{e^{-i\omega\sqrt{2}t}}{2} + \frac{e^{i\omega 2t}}{2}\right) & -\left(\frac{e^{-i\omega\sqrt{2}t}}{2} + \frac{e^{i\omega\sqrt{2}t}}{2}\right) \\ & & -i\sqrt{2}\left(\frac{e^{-i\omega\sqrt{2}t}}{2} - \frac{e^{+i\omega\sqrt{2}t}}{2}\right) \\ & & \frac{e^{-i\omega\sqrt{2}t}}{2} + \frac{e^{i\omega\sqrt{2}t}}{2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0 & 1\\0 & 0 & 0\\1 & 0 & 1\end{pmatrix} + \frac{1}{2}\begin{pmatrix} \cdots & -\cos\sqrt{2}\omega t \\ & -\sqrt{2}\sin\sqrt{2}\omega t \\ \cdots & \cos\sqrt{2}\omega t \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} \cdots & 1-\cos\sqrt{2}\omega t \\ \cdots & -\sqrt{2}\sin\sqrt{2}\omega t \\ \cdots & 1+\cos\sqrt{2}\omega t \end{pmatrix}$$

$$|\psi(0)\rangle = \frac{1}{2}\begin{pmatrix} \cdots & 1-\overset{0}{\underline{\cos\sqrt{2}\omega t}} \\ \cdots & -\overset{0}{\underline{\sqrt{2}\sin\sqrt{2}\omega t}} \\ \cdots & 1+\cos\sqrt{2}\omega t \end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$