

$$\int \frac{dx}{x^2 - a^2}$$

~~$x = a \sin \theta$~~

$x = a \sec \theta$

$dx = a \sec \theta \tan \theta d\theta$

$$\int \frac{a \sec \theta \tan \theta d\theta}{a^2 \sec^2 \theta - a^2} = \int \frac{a \sec \theta \tan \theta d\theta}{a^2 (\sec^2 \theta - 1)}$$

$$\frac{1}{a} \int \frac{\sec \theta \tan \theta}{\tan^2 \theta} d\theta = \frac{1}{a} \int \frac{\sec \theta}{\tan \theta} d\theta$$

$$= \frac{1}{a} \int \frac{\frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} d\theta = \frac{1}{a} \int \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} d\theta$$

$$= \frac{1}{a} \int \frac{1}{\sin \theta} d\theta = \frac{1}{a} \int \csc \theta d\theta$$

~~$y = \csc x$~~

~~$\frac{dy}{dx} = \frac{d}{dx}(\csc x) = \frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{d}{dx}(\sin x)^{-1}$~~

~~$= \frac{1 \cdot \cos x - \sin x \cdot 0}{\sin^2 x} = \frac{-\cos x}{\sin^2 x}$~~

~~$= \frac{-\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \csc x$~~