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EE372 Challenge: The following problem is open to everyone in the Spring-2011 Class. Anyone submitting a correct solution before the last day of instruction (May 5, 2011) will get 10 points in the final grade and the first one submitting the correct solution will win \$50.

Let $\varepsilon = \varepsilon\,(x, y, z)$ and $\mu = \mu\,(x, y, z)$ for an inhomogeneous medium. The wave equation for electric field in this medium can be derived as

$$\nabla^2 \vec{E} + k_0^2 n^2 \vec{E} + \nabla(\ln \mu) \times \left(\nabla \times \vec{E}\right) + \nabla\left(\nabla \ln \varepsilon \cdot \vec{E}\right) = 0 \tag{i}$$

where $n = \sqrt{\mu_r \varepsilon_r}$ is the index of refraction of the medium, $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ is the wave number in free space and is constant. Assume

$$\vec{E}(\vec{r}) = \vec{e}(\vec{r}) e^{-jk_0 S(\vec{r})}$$

where $S(\vec{r})$ is a scalar function of location $\vec{r}$. Prove that the wave equation (i) can be expressed in terms of $S(\vec{r})$:

$$\begin{aligned}\left(|\nabla S|^2 - k_0^2 n^2\right)\vec{e} = {} & \frac{1}{jk_0}\left\{\left[\nabla^2 S - \nabla(\ln \mu)\cdot \nabla S\right]\vec{e} + 2(\nabla S \cdot \nabla)\vec{e} + 2\nabla S(\vec{e} \cdot \nabla \ln n)\right\} \\ & - \frac{1}{(jk_0)^2}\left[\nabla^2 \vec{e} + \nabla(\vec{e} \cdot \nabla \ln \varepsilon) - (\nabla \times \vec{e}) \times \nabla(\ln \mu)\right]\end{aligned} \tag{ii}$$

Note: You can discuss this problem with anyone or consult Internet to get ideas, but do not copy their solutions.