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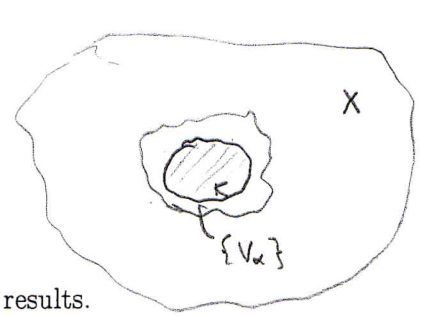
2. Let (X, d) be a metric space and K a compact subset of X.

(a) [20] Prove that K is closed;

(b) [15] If $F \subseteq K$ and F is closed, prove that F is compact.



Please use the ***open cover*** definition of compactness to prove these results.

Proof: (a) If $K \subseteq X$ is compact, K is closed. Since K compact, let $\{V_\alpha\}$ be an open cover. Then since K is compact, it has at least one limit point. So select $p_i \in K'$, $i = 1, \ldots, n$, where n could be infinite. Then Let $\bigcup_{\alpha=i_1}^{i_n} V_\alpha$ be the finite subcover guaranteed by compactness. So, This cover contains all $p_i \in K'$. But all $p_i \in K$ itself, so K closed.

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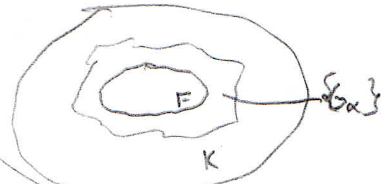
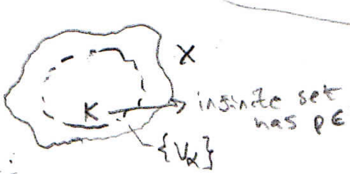
(b)



Let $\{G_\alpha\}$ be an open cover for closed F. Let $\{G_\alpha\}$ adjoined with F^c be an open cover for compact K. Call it Ω. Then, since K has a finite subcover, let Φ be the union of an open set G_* covering F and $\{G_\alpha\} - G_*$. Thus F is covered by a finite number of subcovers; two in our case.



Try this: Or try K^c is open?? Let $p \in K^c$. Then, let $\{W_\alpha\}$ be open cov. for K.

Can't choose subcovers like this → Choose finite W_{α_i} st $W_{\alpha_i} = N_r(q_1) \cup \cdots \cup N_r(q_i)$, $q_i \in K'$. Then $(V_{\alpha_i})^c = K^c$ only, so for some $r > 0$, $d(p, q_i) > 0$.

Proof: (a) Let (X, d) be a M.S. and $K \subseteq X$ compact. We show K is closed by showing K^c is open. First, choose any $p \in K^c$. Next, since K is compact; pick an open cover $\{W_\alpha\}$. By compactness, extract a finite subcover of K, $\{W_{\alpha_i}\}$ where $W_{\alpha_i} = \bigcup_{i=1}^{n} N_r(q_i)$ with $q_i \in K'$ and $r > 0$. Since $(W_{\alpha_i})^c = K^c$, for some $r > 0$, $d(p, q_i) > 0$ for all $p \in K^c$, $q_i \in K'$. Thus for $s > 0$, $\exists N_s(p) \subseteq K^c$. So K^c is open.

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(b) If $F \subseteq K$, F closed, K compact, in (X, d), we show F is compact. Let $\{G_\alpha\}$ be an open cover for F. Adjoin $\{G_\alpha\}$ with F^c to be an open cover for compact K. Call it Ω. But since K is compact, it has a finite subcover of Ω called ${}^2\Phi$, where $\Phi = \{G_*\} \cup (\{G_\alpha\} - G_*)$ (↑ G star) where G_* covers F. Thus F is covered by a finite number of subcovers. Thus F is compact.

How can you arbitrarily make open covers to be balls.

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12 ⇒ Need to pull out F^c. You are not quite clear about the proof.