Ricci Tensor - the amount by which the volume element of a geodesic ball in a curved Riemannian manifold deviates from the standard ball in Euclidean space.

$T_{ss} \approx T_{expl}$

T_{hydr} hydrostatic time-scale

$$T_{hydr} \approx \left(\frac{R}{\frac{GM}{R^2}}\right)^{\frac{1}{2}} \approx \left(\frac{R^3}{GM}\right)^{\frac{1}{2}}$$

2.5 Non-Spherical Case

$$\left. e\frac{dv}{dt} = -\nabla P - e\nabla\Phi \quad (r(v,t)) \right] \text{Eulerian} \qquad -\frac{1}{r^2} = -\nabla$$

V is the velocity vector, and substantial time derivative on the left is defined by the operator:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + v \cdot \nabla$$

The general form of ~~[illegible]~~ has already been shown to be the continuity equation of hydrodynamics:

$$\frac{\partial e}{\partial t} = -\nabla \cdot (eV)$$

Poisson equation ~~[illegible]~~ - gravitational potential (Φ) is connected with an arbitrary distribution of the density:

$$\nabla^2 \Phi = 4\pi G e$$

2.6 Hydrostatic Equilibrium in General Relativity

Einstein field equations:

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{K}{c^2} T_{ik}, \quad K = \frac{8\pi G}{c^2}$$

R_{ik} = Ricci Tensor
g_{ik} = metric tensor
R = Riemann curvature
T_{ik} = Energy-momentum tensor

$T_{00} = ec^2$, $T_{11} = T_{22} = T_{33} = P$ (e includes the energy density, P = pressure)

Line element ds (distance between two neighbouring events in given spherical coordinates r, ϑ, φ) (time-independent)

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2)$$

with $v = v(r)$, $\lambda = \lambda(r)$. Expressions T_{ik} and ds, the field (2.24) equations can be reduced to 3 ordinary differential equations:

$$\frac{KP}{c^2} = e^{-\lambda}\left(\frac{v'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2},$$

Primes denote derivatives with respect to r

$$\frac{KP}{c^2} = \frac{1}{2}e^{-\lambda}\left(v'' + \frac{1}{2}v'^2 + \frac{v' - \lambda'}{r} - \frac{v'\lambda'}{2}\right),$$

$$K\rho = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

Multiply $4\pi r^2$ with $= Km = 4\pi r(1 - e^{-\lambda})$

Here m denotes the "gravitational mass" inside r:

$$m = \int_0^r 4\pi r^2 \rho \, dr \qquad \rho = \rho_0 + \frac{U}{c^2}$$

U = whole energy density
ρ_0 = rest-mass density

the changed metric would give the spherical volume element as $e^{\frac{\lambda}{2}} \cdot 4\pi r^2 dr$ instead of the usual form $4\pi r^2 dr$

Differentiation of $\frac{KP}{c^2} = e^{-\lambda}\left(\frac{v'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$ (with respect to r) gives $P' = P'(\lambda, \lambda', v', v'', r)$ where λ', v', v'' are arrives at the Tolman-Oppenheimer-Volkoff (TOV) equation for hydrostatic equilibrium in general relativity:

How does one come to this equation

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1 + \frac{P}{\rho c^2}\right)\left(1 + \frac{4\pi r^3 P}{mc^2}\right)\left(1 - \frac{2Gm}{rc^2}\right)^{-1} \quad (2.4)$$

$$c^2 \to \infty$$

$$ds^2 = e^v c^2 dt^2 - e^\lambda dr^2 - r^2(d\vartheta + \sin^2\vartheta \, d\varphi^2)$$

$T_{ik} = T_{00} = \rho c^2$, $T_{11} = T_{22} = T_{33} = P$

$R = \frac{K}{c^2} T_{ik}$

$v = v(r)$, $\lambda = \lambda(r)$ $\frac{KP}{c^2} = e^v \cancel{c^2} dt^2 - e^\lambda dr^2 - r^2(d\vartheta + \sin^2\vartheta \, d\varphi^2) \longrightarrow$ with respect to r

$$\frac{KP}{c^2} = e^v - \frac{dt^2}{} + \frac{e^{-\lambda}}{r^2} - \frac{1}{r^2}\left(\frac{d\vartheta}{r} + \frac{\sin\vartheta \, d\varphi}{r^2}\right)$$

$$\frac{KP}{c^2} = e^{-\lambda}\left(\frac{v'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$