h) $\frac{d|\psi\rangle}{dt} = -i\frac{\hat{H}}{\hbar}|\psi\rangle \qquad |\psi\rangle = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = -i\omega \hat{L}_y \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -i\omega \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -i\omega \begin{pmatrix} -iy \\ xi - zi \\ iy \end{pmatrix} = \omega \begin{pmatrix} -y \\ x - z \\ y \end{pmatrix}$$

$\dot{x} = -\omega y$

$\dot{y} = \omega(x - z)$

$\dot{z} = \omega y$

$\ddot{y} = \omega\dot{x} - \omega\dot{z} = -\omega^2 y - \omega^2 y = -2\omega^2 y$

$y'' + 2\omega^2 y = 0$

$r^2 + 2\omega^2 = 0$

$r = \pm\sqrt{2}\,\omega i$

$y(t) = Ae^{\omega i\sqrt{2}t} + Be^{-\omega\sqrt{2}t} = 0.$

$y(0) = A + B = 0$

$\dot{y}(t) = \sqrt{2}i\omega A e^{i\sqrt{2}\omega t} - \sqrt{2}i\omega B e^{-i\sqrt{2}\omega t}$

$y(0) = A - B = 0$

$\dot{z} = \omega y = \omega A e^{i\omega t\sqrt{2}} + \omega B e^{-i\omega t\sqrt{2}} \qquad \dot{z}(0) = \omega A + \omega B = 0$

$A + B = 0$

$z = -\frac{i}{\sqrt{2}} A e^{i\omega t\sqrt{2}} + \frac{i}{\sqrt{2}} B e^{-i\omega t\sqrt{2}}$

$z(0) = -\frac{i}{\sqrt{2}}A + \frac{i}{\sqrt{2}}B = 1$

$-A + B = \frac{\sqrt{2}}{i}$ → $2B = \frac{\sqrt{2}}{i}$ → $B = -\frac{\sqrt{2}}{2}i$

$A = \frac{\sqrt{2}}{2}i$

$\Rightarrow z(t) = -\frac{1}{2}e^{i\omega t\sqrt{2}} + \frac{1}{2}e^{-i\omega t\sqrt{2}} = \cos\sqrt{2}\omega t$

$y = \frac{\dot{z}}{\omega} = Ae^{i\omega t\sqrt{2}} + Be^{-i\omega t\sqrt{2}} = \frac{\sqrt{2}}{2}ie^{i\omega t\sqrt{2}} - \frac{\sqrt{2}}{2}ie^{-i\omega t\sqrt{2}}$

$= -\frac{\sqrt{2}\omega \sin\sqrt{2}\omega t}{\omega} = -\sqrt{2}\sin\sqrt{2}\omega t$

$\dot{x} = -\omega y = \omega\sqrt{2}\sin\sqrt{2}\omega t$

$x(t) = -\cos(\sqrt{2}\omega t) + c = -\cos\sqrt{2}\omega t + 1$

$$|\psi(t)\rangle = -\cos\sqrt{2}\omega t + 1\,|v_1\rangle + -\sqrt{2}\sin\sqrt{2}\omega t\,|v_2\rangle + \cos\sqrt{2}\omega t\,|v_3\rangle$$