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**Hint:** The following identities may be useful. Note that  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  are vectors and  $\phi$  and  $\psi$  are scalars.

$$\nabla(\phi\psi) = \phi\nabla\psi + \psi\nabla\phi \quad (1)$$

$$\nabla \cdot (\psi\vec{A}) = \nabla\psi \cdot \vec{A} + \psi\nabla \cdot \vec{A} \quad (2)$$

$$\nabla \times (\psi\vec{A}) = \nabla\psi \times \vec{A} + \psi\nabla \times \vec{A} \quad (3)$$

$$\nabla(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \quad (4)$$

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B} \quad (5)$$

$$\vec{A} \times \vec{B} \times \vec{C} = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C} \quad (6)$$

$$\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \quad (7)$$

$$\nabla \times (\nabla\phi) = 0 \quad (8)$$

The Laplacian operator  $\nabla^2$  is defined as

$$\nabla^2\phi = \nabla \cdot (\nabla\phi) \quad (9)$$

Note that  $\nabla^2$  can also operate on a vector. In this case,  $\nabla^2$  will operate on each of the components of the vector and the result is also a vector.

For a scalar function  $S = S(\vec{r})$  and a constant  $k_0$ , we have

$$\nabla e^{-jk_0 S} = -jk_0 (\nabla S) e^{-jk_0 S} \quad (10)$$

If  $u = u(\vec{r})$  is a scalar function, it is not difficult to derive that

$$\nabla(\ln u) = \frac{1}{u} \nabla u \quad (11)$$

$$\nabla\left(\frac{1}{u}\right) = -\frac{1}{u} \nabla(\ln u) \quad (12)$$