

$$\nabla^2 \vec{E} + k^2 n^2 \vec{E} + \nabla(\ln \mu) \times (\nabla \times \vec{E}) + \nabla(\nabla \ln \epsilon \cdot \vec{E}) = 0 \quad *$$

since  $\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$  (7)

$$\nabla^2 \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla \times \nabla \times \vec{A} \quad (\text{rearranging (7)})$$

$$\therefore \nabla^2 \vec{E} = \nabla(\nabla \cdot \vec{E}) - \nabla \times \nabla \times \vec{E}$$

since  $\nabla \ln \epsilon$  gives a vector ( $\ln \epsilon$  is scalar), we can use

$$\nabla(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}) \quad (4)$$

in the following expression:  $\nabla(\nabla \ln \epsilon \cdot \vec{E})$ , which turns to

$$\nabla(\nabla \ln \epsilon \cdot \vec{E}) = (\nabla \ln \epsilon \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \nabla \ln \epsilon + \nabla \ln \epsilon \times (\nabla \times \vec{E}) + \vec{E} \times (\nabla \times \nabla \ln \epsilon)$$

The last term,  $\vec{E} \times (\nabla \times \nabla \ln \epsilon)$ , can be neglected since

$$\nabla \times (\nabla \phi) = 0 \quad \text{where } \phi \text{ is a scalar (like } \ln \epsilon) \quad (8)$$

$$\therefore \nabla(\nabla \ln \epsilon \cdot \vec{E}) = (\nabla \ln \epsilon \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \nabla \ln \epsilon + \nabla \ln \epsilon \times (\nabla \times \vec{E})$$

Next, consider  $\nabla(\ln \mu) \times (\nabla \times \vec{E})$

$\nabla \ln \mu$  gives a vector, so we have:

$$\vec{A} \times (\nabla \times \vec{B}) \quad \text{where } \vec{A} = \nabla \ln \mu \text{ and } \vec{B} = \vec{E}$$

so we can use (4) again,

$$\nabla(\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A})$$

and rearrange to solve for  $\vec{A} \times (\nabla \times \vec{B})$

$$\vec{A} \times (\nabla \times \vec{B}) = \nabla(\vec{A} \cdot \vec{B}) - (\vec{A} \cdot \nabla) \vec{B} - (\vec{B} \cdot \nabla) \vec{A} - \vec{B} \times (\nabla \times \vec{A})$$

$$\therefore \nabla(\ln \mu) \times (\nabla \times \vec{E}) = \nabla(\nabla \ln \mu \cdot \vec{E}) - (\nabla \ln \mu \cdot \nabla) \vec{E} - (\vec{E} \cdot \nabla) \nabla \ln \mu - \vec{E} \times (\nabla \times \nabla \ln \mu)$$