

$\therefore$  \* turns to:

$$13) \nabla(\nabla \cdot \vec{E}) - \nabla \times \nabla \times \vec{E} + k_0^2 n^2 \vec{E} + \nabla(\nabla \ln \mu \cdot \vec{E}) - (\nabla \ln \mu \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \nabla \ln \mu + \vec{E} \times (\nabla \times \nabla \ln \mu) + (\nabla \ln \epsilon \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \nabla \ln \epsilon + \nabla \ln \epsilon \times (\nabla \times \vec{E}) = 0$$

To simplify the expression in order to identify usable identities,  
let  $\nabla \ln \mu = \vec{A}$  and  $\nabla \ln \epsilon = \vec{B}$

$$14) \nabla(\nabla \cdot \vec{E}) - \nabla \times \nabla \times \vec{E} + k_0^2 n^2 \vec{E} + \nabla(\vec{A} \cdot \vec{E}) - (\vec{A} \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \vec{A} + \vec{E} \times (\nabla \times \vec{A}) + (\vec{B} \cdot \nabla) \vec{E} + \underline{(\vec{E} \cdot \nabla) \vec{B}} + \vec{B} \times (\nabla \times \vec{E}) = 0$$

since we have  $(\vec{E} \cdot \nabla) \vec{B}$ , we can use (5) rearranged:

$$(\vec{E} \cdot \nabla) \vec{B} = \vec{E}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{E}) + (\vec{B} \cdot \nabla) \vec{E} - \nabla \times (\vec{E} \times \vec{B})$$

$\therefore$  13) becomes:

$$15) \nabla(\nabla \cdot \vec{E}) - \nabla \times \nabla \times \vec{E} + k_0^2 n^2 \vec{E} + \nabla(\vec{A} \cdot \vec{E}) - (\vec{A} \cdot \nabla) \vec{E} + (\vec{E} \cdot \nabla) \vec{A} + \vec{E} \times (\nabla \times \vec{A}) + (\vec{B} \cdot \nabla) \vec{E} + \vec{E}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{E}) + (\vec{B} \cdot \nabla) \vec{E} - \nabla \times (\vec{E} \times \vec{B}) + \vec{B} \times (\nabla \times \vec{E}) = 0$$

Now, using (2)  $\nabla \cdot (\gamma \vec{A}) = \nabla \gamma \cdot \vec{A} + \gamma \nabla \cdot \vec{A}$ , we can expand as follows:

$$\vec{E}(\vec{r}) = \vec{e}(\vec{r}) e^{-jk_0 s(\vec{r})}, \quad \text{let } \gamma = e^{-jk_0 s(\vec{r})} \text{ and } \vec{A} = \vec{e}(\vec{r})$$

$$\therefore \nabla \cdot \vec{E} = \nabla e^{-jk_0 s(\vec{r})} \cdot \vec{e}(\vec{r}) + e^{-jk_0 s(\vec{r})} \nabla \cdot \vec{e}(\vec{r})$$

$$\text{We also know that } \nabla e^{-jk_0 s} = -jk_0 (\nabla s) e^{-jk_0 s}$$

$$\therefore \nabla \cdot \vec{E} = -jk_0 (\nabla s) e^{-jk_0 s} \cdot \vec{e} + e^{-jk_0 s} \nabla \cdot \vec{e}$$

using (3),  $\nabla \times (\gamma \vec{A}) = \nabla \gamma \times \vec{A} + \gamma \nabla \times \vec{A}$ , we can also expand as follows:

$$\nabla \times \vec{E} = \nabla e^{-jk_0 s} \times \vec{e} + e^{-jk_0 s} \nabla \times \vec{e}$$

$$\therefore \nabla \times \vec{E} = -jk_0 (\nabla s) e^{-jk_0 s} \times \vec{e} + e^{-jk_0 s} \nabla \times \vec{e}$$