Determining The Inclination Angle of Spectroscopic Binaries

Akshat Tripathi

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The true velocity of a star in the binary system is shown by V_r, the inclination angle of the system is i when viewed from pos $P1$ (on one side of the sun) and the component of the stars velocity that we see on earth is $V_r sin(i)$ When we look at the same system from a different side of the sun, the inclination angle becomes $i + di$ and the component of the star's velocity that we see on earth is $V_r sin(i + di)$. In addition, the change in the inclination angle (di) that happens due to earth's movement around is the same angle that is formed between the two lines joining the two different positions of the earth to the binary system.

Now, let:

$$V_r sin(i) = a \quad (1)$$

and

$$V_r sin(i + di) = b \quad (2)$$

Using trig identity, we can expand (2) and write:

$$V_r sin(i)cos(di) + cos(i)sin(di) = b$$

$$=> V_r[\frac{a}{V_r} cos(di) + \frac{\sqrt{V_r^2 - a^2}}{V_r} sin(di)] = b$$

$$=> acos(di) + \sqrt{V_r^2 - a^2} sin(di) = b$$

$$=> \sqrt{V_r^2 - a^2} = \frac{b - acos(di)}{sin(di)}$$

$$=> V_r^2 = \left(\frac{(b - acos(di))}{sin(di)}\right)^2 + a^2$$

$$=> V_r = \sqrt{\left(\frac{(b - acos(di))}{sin(di)}\right)^2 + a^2}$$

Hence,

$$=> i = sin^{-1} \frac{a}{\sqrt{\left(\frac{(b-acos(di))}{sin(di)}\right)^2 + a^2}}$$

While this formula seems simple enough, I am concerned that measuring the change in inclination angle, di, will pose a challenge. But with the Gaia Spacecraft, it should not be too difficult measuring that angle since it can measure parallax angles to a precision of 20 microarcseconds. The other problem that might arise is measuring the two velocity components a and b.

One of the most sensitive spectrometers in the world, the one installed in the Very Large Telescope in the European Space Observatory has a resolution of 10^5 :

$$R = \frac{\lambda}{\Delta\lambda} = 10^5$$

Given that redshift is:

$$z = \frac{v}{c} = \frac{\Delta\lambda}{\lambda}$$

Now, if I plug in the resolution value in the redshift equation and the speed of light, I get a number for the velocity. However, I am not sure what that number means exactly. Also, I am having trouble figuring out if with this sensitivity, we will be able to detect a small change in velocity due to the small change in angle or not.