real analysis exercises :

$a_1, a_2, a_3, \ldots, a_n$ *are arbitrary real numbers, prove that* :

$$\left|\sum_{i=1}^{n} a_i\right| \le \sum_{i=1}^{n} |a_i|$$

I want to prove the above statement by induction. First i establish a point, where i know the statement is true:

n=0:

$0 \le 0$

$n = 1$

$|a_1| \le |a_1|$

It seems that the statement may be true for n>0. Now i assume that it is valid for some n=k. I want to show that it is indeed valid for n=k+1:

$$\left|\sum_{i=1}^{k} a_i\right| + |a_{k+1}| \le \sum_{i=1}^{k+1} |a_i| \quad \text{(i)}$$

i know that the following is valid for n=k (inductive hypothesis?):

$$\left|\sum_{i=1}^{k} a_i\right| \le \sum_{i=1}^{k} |a_i|$$

If i can reduce (i) to something i know is true, then (i) must be true:

$$\left|\sum_{i=1}^{k} a_i\right| \le \sum_{i=1}^{k+1} |a_i| - |a_{k+1}|$$

$$\left|\sum_{i=1}^{k} a_i\right| \le \sum_{i=1}^{k} |a_i|$$

Which i know for a fact to be true. It is therefore shown, that the statement is true for all n>0.