Proposition: $x_1, x_2, \dots, x_n \in [0,1]$

Then $* \prod_{i=1}^{n} (1-x_i) \geq 1 - \sum_{i=1}^{n} x_i$

induction on n.

Basis case:

$n=1$ $L-H-S = 1-x_1$

$R-H-S = 1-x_1$ ✓

Inductive hypothesis:

Assume $(*)$ is true for $n \geq 2$

now we show that $*$ holds for $n+1$

1) So, $\prod_{i=1}^{n} (1-x_i) = \prod_{i=1}^{n} (1-x_i)(1-x_{n+1})$

2) $\overset{IH}{\geq} \left(1 - \sum_{i=1}^{n} (x_i)(1-x_{n+1})\right)$

3) $= 1 - \sum_{i=1}^{n} x_i - x_{n+1} + x_{n+1} \sum_{i=1}^{n} x_i$

4) $= 1 - \sum_{i=1}^{n+1} x_i + x_{n+1} \sum_{i=1}^{n} x_i$

5) $\geq 1 - \sum_{i=1}^{n} x_i$, since $x_i \in [0,1]$