$with(IntegrationTools):$

$$\int_0^{2\pi} e^{-I\cdot k\sqrt{r^2-2\cdot a\cdot r\cdot \sin(\theta)\cdot\cos(\phi)+a^2}}\,d\phi$$

$$\sqrt{r^2+a^2-2\cdot a\cdot r\cdot \sin(\theta)\,\cos(\phi)} \xrightarrow{\text{differentiate w.r.t. phi}} \frac{a\,r\sin(\theta)\,\sin(\phi)}{\sqrt{r^2+a^2-2\,a\,r\sin(\theta)\,\cos(\phi)}}$$

$$\sqrt{r^2+a^2-2\cdot a\cdot r\cdot \sin(\theta)\,\cos(\phi)}=u \xrightarrow{\text{solutions for phi}} \arccos\left(\frac{1}{2}\,\frac{r^2+a^2-u^2}{a\,r\sin(\theta)}\right)$$

$$\int e^{-I\cdot k\cdot u}\,\frac{u}{a\,r\sin(\theta)\,\sin\left(\arccos\left(\frac{1}{2}\,\frac{r^2+a^2-u^2}{a\,r\sin(\theta)}\right)\right)}\,du$$

$$\int \frac{2\,e^{-Iku}\,u}{a\,r\sin(\theta)\,\sqrt{4-\frac{\left(r^2+a^2-u^2\right)^2}{a^2\,r^2\sin(\theta)^2}}}\,du \tag{1}$$

$$\sin\left(\arccos\left(\frac{1}{2}\,\frac{r^2+a^2-u^2}{a\,r\sin(\theta)}\right)\right)$$

$$\frac{1}{2}\,\sqrt{4-\frac{\left(r^2+a^2-u^2\right)^2}{a^2\,r^2\sin(\theta)^2}} \tag{2}$$