(c) Discuss the pointwise, uniform, and L^2 convergence of the full Fourier series of $\Lambda(x)$.

(d) If they exist, find the value of the full Fourier series for $\Lambda(x)$ at $x = 100$ and $x = 101.5$.

(e) Can the Fourier series for $\Lambda(x)$ be differentiated term-by-term? If so, where? Discuss.

9. Consider the function

$$f(x) = \begin{cases} e^x, & -1 \leq x \leq 0, \\ mx + b, & 0 \leq x \leq 1. \end{cases}$$

Without computing any Fourier coefficients, answer the following.

(a) For what value(s) of m and b (if any) will the full Fourier series of $f(x)$ converge pointwise on $-1 < x < 1$? Justify your answer.

(b) For what value(s) of m and b (if any) will the full Fourier series of $f(x)$ converge uniformly on $-1 \leq x \leq 1$? Justify your answer.

(c) For what value(s) of m and b (if any) will the full Fourier series of $f(x)$ converge in the L^2 sense on $-1 \leq x \leq 1$? Justify your answer.

❄ 10. Let $f(x) = x$, $-1 < x < 1$. Plot the $N = 10$ partial Fourier sum for f, as well as the term-by-term antiderivative and term-by-term derivative. Explain your results in light of Theorems 3.5 and 3.6.

11. **(Weierstrass M-Test for Uniform Convergence)** Another way to prove uniform convergence of $\sum_{n=1}^{\infty} c_n X_n(x)$, $a \leq x \leq b$, is the following result.

If $\{M_n\}_{n=1}^{\infty}$ are positive constants such that $|c_n X_n(x)| \leq M_n$ for all x in $[a, b]$ and $\sum_{n=1}^{\infty} M_n < \infty$, then $\sum_{n=1}^{\infty} c_n X_n(x)$ converges uniformly on $[a, b]$.

Use this to answer the following.

(a) Use the Weierstrass M-Test to show that the Fourier series for $|x|$, $-\pi \leq x \leq \pi$, given in Exercise 6 converges uniformly on $-\pi \leq x \leq \pi$.

(b) More generally, use the Weierstrass M-Test to show that if a_n and b_n are the Fourier coefficients in (3.8) and $\sum_{n=1}^{\infty}(|a_n| + |b_n|) < \infty$, then (3.8) converges uniformly on $-\ell \leq x \leq \ell$.

(c) Suppose $f(x)$, $-\ell \leq x \leq \ell$, has Fourier coefficients

$$a_0 = \frac{2}{3}, \quad a_n = \frac{4(-1)^n}{\pi^2 n^2}, \quad b_n = 0, \quad n = 1, 2, \ldots.$$

Does the full Fourier series of $f(x)$ converge uniformly on $[-\ell, \ell]$? Explain.