

Relativistic Mass

Karl -

You have an article (see URL above) on the derivation of $E = mc^2$ which requires the veracity of the relativistic mass equation:

$$m = \frac{m_0}{\text{SQRT}[1 - (v^2)/(c^2)]}$$

m = mass in motion; m_0 = mass at rest, v = velocity in motion, c = speed of light as constant 300,000 km/sec

I have never seen the derivation of this relativistic mass equation.

Can you please point me the way? No Internet site has it.

Also, you can't use photons or electromagnetic data to prove it. That was how Maxwell did it but there is no proof that it is true for all phenomena.

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First you must establish time dilation from the first principle that the speed of light is constant whether you are moving toward or away from the source or whether the source is moving toward or away from you.

Imagine a man in a train reading a book. There is a reading light one meter directly above his book. The time he observes for the light from the reading light to reach his book is $t = 1 \text{ meter} / c$.

Suppose the train moves past the station at velocity, v . An observer on the platform sees the light moving not directly down, but at an angle forward. So the light traverses a longer distance from this observer's point of view. Since the light is still traveling at c , the time the platform observer sees for the light to travel from the light to the book is longer. Call that time, t' , which is greater than t .

In time, t' , the train moves vt' horizontally. The total distance the light travels from the platform observer's point of view, according to the Pythagorean formula:

$$s = \text{SQRT}((1 \text{ meter})^2 + (vt')^2)$$

Total time of travel is $t' = s/c$

$$t' = s/c = \text{SQRT}((1 \text{ meter})^2 / c^2 + (vt')^2 / c^2)$$

Observe that $1 \text{ meter} / c = t$

$$t' = s/c = \text{SQRT}(t^2 + (vt')^2 / c^2)$$

Squaring both sides:

$$t'^2 = t^2 + v^2 t'^2 / c^2$$

$$t'^2 (1 - v^2 / c^2) = t^2$$

$$t' = t / \text{SQRT}(1 - v^2 / c^2)$$

Now suppose the observer on the train has a bowling ball. A man standing in the door of the coach has an identical bowling ball. They both throw their bowling balls perpendicular to the motion of the train, both at the velocity they observe as u (which we assume is much less than v). That is the platform observer sees his bowling ball moving toward the train at velocity, u , and the train observer sees his bowling ball moving toward the platform at velocity, u .

The two bowling balls collide elastically and each bounces directly back to its thrower. Physics requires momentum to be conserved. The platform observer sees the lateral velocity of the train observer's bowling ball as moving less than u , and likewise the train observer sees the platform observer's bowling ball as moving less than u . By symmetry, the platform observer sees his bowling ball returning to him from the collision at u . Likewise with the train observer.

Momentum is mass times velocity. Momentum has both a lateral component and a longitudinal component. Each component must be conserved. Since the platform observer sees the train observer's ball moving laterally at a slower rate than his own, in order for both balls to have equal but opposite momentum, the platform observer must see the train observer's ball as having greater mass than his own. Likewise with the train observer's view of the mass of the platform observer's bowling ball.

When you work it out, if m_p is the mass of the platform observer's ball (from his point of view), he must see the train observer's ball as having mass

$$m_p / \text{SQRT}(1 - v^2 / c^2)$$

for momentum to be preserved.

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