Example 26.4 Charging a battery

In the circuit shown in Fig. 26.11, a 12-V power supply with unknown internal resistance r is connected to a run-down rechargeable battery with unknown emf $\mathcal{E}$ and internal resistance 1 Ω and to an indicator light bulb of resistance 3 Ω carrying a current of 2 A. The current through the run-down battery is 1 A in the direction shown. Find the unknown current I, the internal resistance r, and the emf $\mathcal{E}$.

SOLUTION

IDENTIFY and SET UP: We assume the direction of the current through the 12-V power supply to be as shown. This circuit has more than one loop, so we must apply both the junction rule and the loop rule. There are three target variables, so we need three equations.

EXECUTE: First we apply the junction rule, Eq. (26.5), to point *a*. We find

$$-I + 1\text{ A} + 2\text{ A} = 0 \quad \text{so} \quad I = 3\text{ A}$$

To determine r, we apply the loop rule, Eq. (26.6), to the outer loop labeled (1); we find

$$12\text{ V} - (3\text{ A})r - (2\text{ A})(3\ \Omega) = 0 \quad \text{so} \quad r = 2\ \Omega$$

The terms containing the resistances r and 3 Ω are negative because our loop traverses those elements in the same direction as the current and hence finds potential *drops*. If we had chosen to traverse loop (1) in the opposite direction, every term would have had the opposite sign, and the result for r would have been the same.

To determine $\mathcal{E}$, we apply the loop rule to loop (2):

$$-\mathcal{E} + (1\text{ A})(1\ \Omega) - (2\text{ A})(3\ \Omega) = 0 \quad \text{so} \quad \mathcal{E} = -5\text{ V}$$

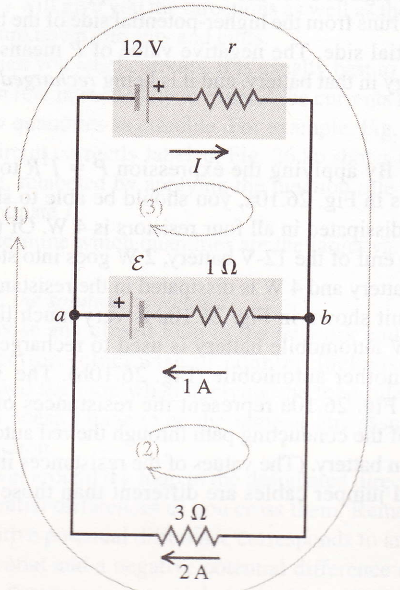
The term for the 1-Ω resistor is positive because in traversing it in the direction opposite to the current we find a potential *rise*. The negative value for $\mathcal{E}$ shows that the actual polarity of this emf is opposite to the assumption made in Fig. 26.11; the positive terminal of this source is really on the right side. As in Example 26.3, the battery is being recharged.

EVALUATE: We can check our result for $\mathcal{E}$ by using loop (3), obtaining the equation

$$12\text{ V} - (3\text{ A})(2\ \Omega) - (1\text{ A})(1\ \Omega) + \mathcal{E} = 0$$

from which we again find $\mathcal{E} = -5$ V.

As an additional consistency check, we note that $V_{ba} = V_b - V_a$ equals the voltage across the 3-Ω resistance, which is $(2\text{ A})(3\ \Omega) = 6$ V. Going from a to b by the top branch, we encounter potential differences $+12\text{ V} - (3\text{ A})(2\ \Omega) = +6$ V, and going by the middle branch we find $-(-5\text{ V}) + (1\text{ A})(1\ \Omega) = +6$ V. The three ways of getting V_{ba} give the same results. Make sure that you understand all the signs in these calculations.



26.11 In this circuit a power supply charges a run-down battery and lights a bulb. An assumption has been made about the polarity of the emf $\mathcal{E}$ of the run-down battery; is this assumption correct?

Example 26.5 Power in a battery-charging circuit

In the circuit of Example 26.4 (shown in Fig. 26.11), find the power delivered by the 12-V power supply and by the battery being recharged, and find the power dissipated in each resistor.

SOLUTION

IDENTIFY and SET UP: We use the results of Section 25.5, in which we found that the power delivered *from* an emf to a circuit is $\mathcal{E}I$ and the power delivered *to* a resistor from a circuit is $V_{ab}I = I^2R$.

EXECUTE: The power output from the emf of the power supply is

$$P_{\text{supply}} = \mathcal{E}_{\text{supply}}I_{\text{supply}} = (12\text{ V})(3\text{ A}) = 36\text{ W}$$