1. INTRODUCTION

1.1 Background

This document provides an independent experimental testing of the principles shown in experimental demonstration and theoretical analysis provided by Prof. Lewin in Lecture 16 of his MIT 8.0.2 Course on Electricity and Magnetism, available on-line at mit.edu or youtube.com. This course is a freshman level introductory course on the subject. Historically, this particular lecture, and the associated experiment presented there, are often misinterpreted. Hence, an independent verification may help some people. In this document more details of a measurement setup are provided, and each measurement has pictures and scope screen-captures. The additional detail may allow lingering questions, that can't be answered by viewing the lectures, to be answered. If any questions do remain, this document can provide an aid for the reader to do their own experiments, while avoiding the common pitfalls that an experienced person could easily hit, and a novice will almost surely hit on a first attempt.

Fig. 1.1 shows the idealized circuit given by Prof. Lewin. A magnetic field is shown in the shaded region. It is perpendicular to the page, and changing in time. The return flux is assumed to circulate around outside of the circuit, and is not shown. Two identical voltmeters V_1 and V_2, and two resistors R_1 and R_2 are in the circuit as shown. Nodes **A** and **D** are identified.

The internal resistance of each voltmeter is assumed to be much greater than R_1 and R_2, and all connecting wires are assumed to have negligible resistance. Of course, these various assumptions are idealizations, and care must be used in any real experiment to make sure that the non-ideal components, and arrangement of them, are suitable to meet the assumptions to high accuracy. Three loops are identified: the left loop with V_1 and R_1, the middle loop with R_1 and R_2, and the right loop with V_2 and R_2. The loop currents are I_1, I and I_2 respectively, as shown in the diagram. One can easily show that Faraday's Law works for all three loops, and Prof. Lewin provides a nice document that goes through this in great detail. It is also possible to identify other loops, and Faraday's Law applies to those as well.

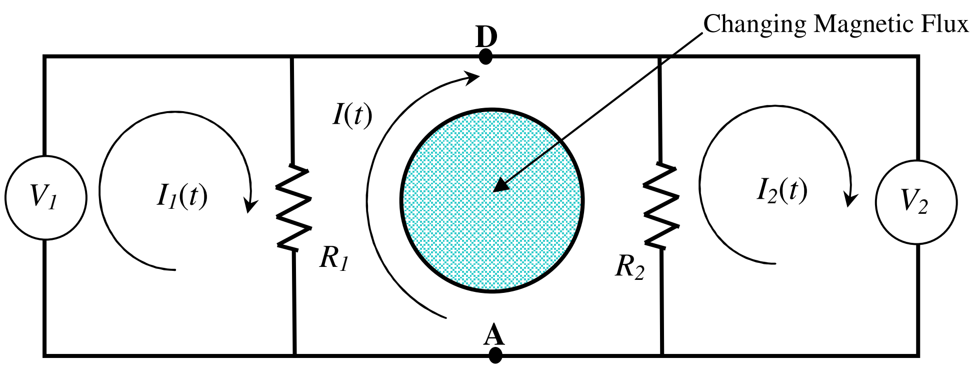

Fig. 1.1 Idealized diagram of Prof. Lewin experiment

1.2 Experimental Setup

Great care must be used to develop a physical setup that meets the assumptions of the idealized circuit above. Fig. 1.2 helps to show the setup used here, which is not very different from Prof. Lewin's experiment, except that a single dual trace oscilloscope is used instead of two isolated oscilloscopes. The difference is important because the grounds on both channels of a dual trace scope are tied together at the scope and this allows another closed path for leakage flux to enter, unless great care is used. Most modern labs have dual trace scopes, and it is more likely someone will try an experiment with one of these.

In this setup, an air core solenoid is wound to generate the magnetic field. The air core implies smaller field, but more predictable behavior. This tradeoff is justified because we want a controlled experiment. The field in the center of the solenoid is used as the main circuit flux for the experiment. The red lines show the circulating magnetic field that leaks out and could potentially corrupt the experiment. The oscilloscope channels are used as the voltmeters. Channel 1 in yellow monitors R_1, while channel 2 in blue monitors R_2. The figure shows the thicker line to indicate the main probe wire, and the thinner wire is the ground lead that comes off the probe and connects in back at node D. The main circuit with R_1 and R_2 is soldered directly around the solenoid to prevent the leakage flux from getting into the circuit. Any leakage flux getting between the solenoid and the main circuit loop would not actually corrupt the experiment at all, but it would partially cancel out the useful flux that is being generated. Since an air core solenoid is used, it's best not to waste useful flux. Note, however, that it is important to keep the leakage flux out of the measurement loops that include the voltmeters. The presence of such leakage flux would violate the initial assumptions of the circuit given in Fig. 1.1.

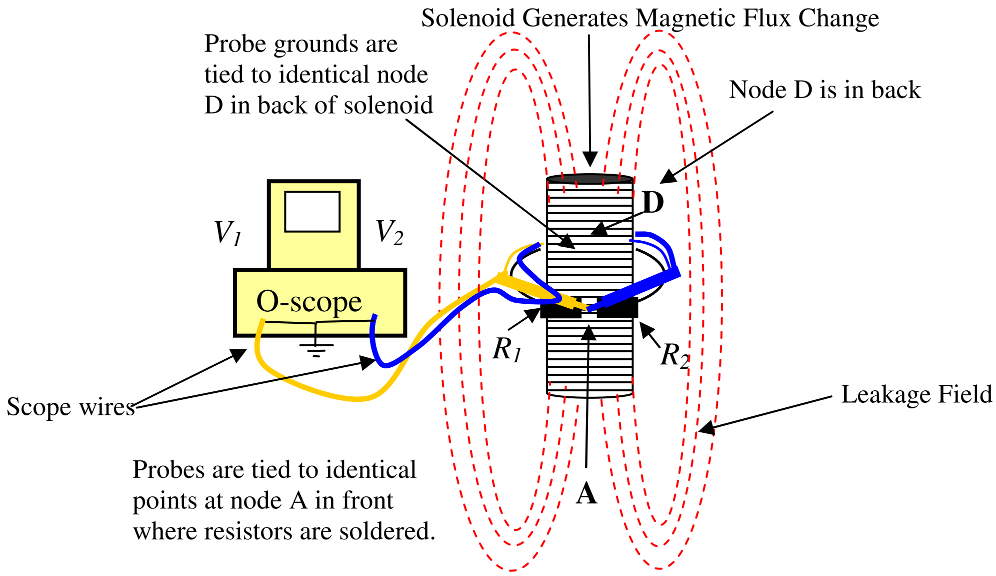

Fig. 1.2 Block diagram of experimental setup

It should be noted that the two scope probes are connected to monitor the **exact same points!** The ground leads are attached at node **D** in the back of the solenoid, and the probe tips are connected at node **A** in the front of the solenoid. Both points are 180 degrees apart on the circle formed by the main circuit loop formed by R_1 and R_2. **One very important precaution** must be made here. That is, the loops formed by the probe wires **must not enclose** any of the leakage flux, **or the measurement will be corrupted**. The probe cable is a coax line that runs the ground and probe wire close together, so very little flux is captured by this. However, the probe, and ground lead near the probe, can form an open loop if care is not taken. To prevent measurement error, the ground and probe should run right along the main circuit loop as shown. This minimizes the area of the open loop and hence minimizes the flux captured by that loop.

The further complications come from the use of a dual channel scope rather than two separate and isolated oscilloscopes. Because of the common ground, the independent probe wires can not run separately, but must overlap, as shown in Fig. 1.2, to prevent creating an open loop that can capture flux. Fig. 1.3 illustrates the wrong method. Here the scope probes are allowed to run separately which opens up a ground loop that will capture a large flux because the area of this loop is very large. The potential mistake is particularly insidious because intuition leads us to try and copy the Lewin arrangement and let the scope wires enter from opposites sides of the circuit. However, it must be remembered that Prof. Lewin is using independent and isolated oscilloscopes as the voltmeters. If we choose to use a dual trace scope for the meters, then extra precautions are needed to make sure the initial circuit assumptions are not violated.

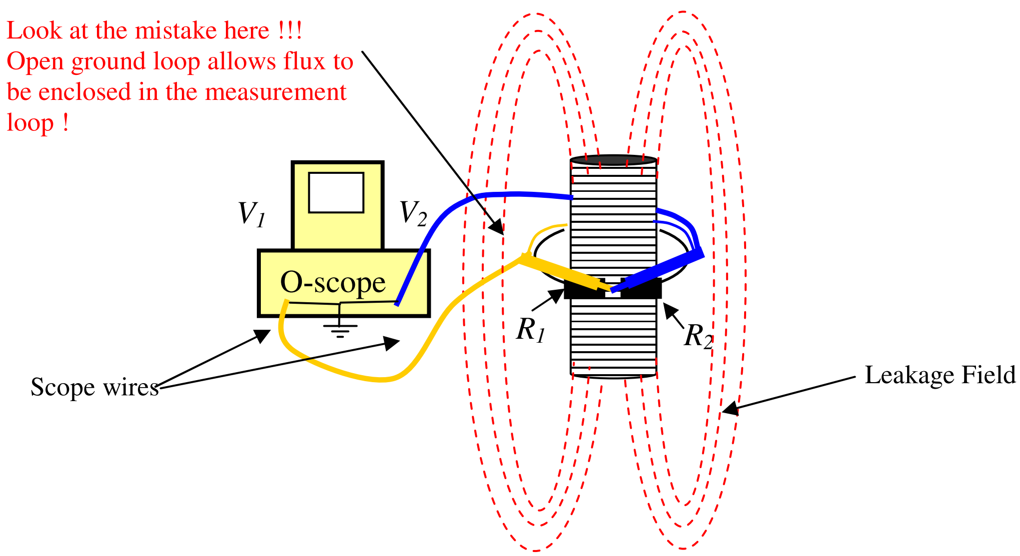

Fig. 1.3 Block diagram of WRONG! experimental setup

1.3 Illustration of Sources of Experimental Error

Fig. 1.4 shows the layout of the wrong way to setup the experiment and illustrates the sources of error that are introduced by improper routing of wires. In this figure, the main measurement loops are open and allow corruption of the measurement with the total flux enclosed. Also, both scope cables are laid in a way that the unintended ground loops capture a large flux, owing to the large area created when the separate channel leads are brought into the measurement zone from opposite sides.

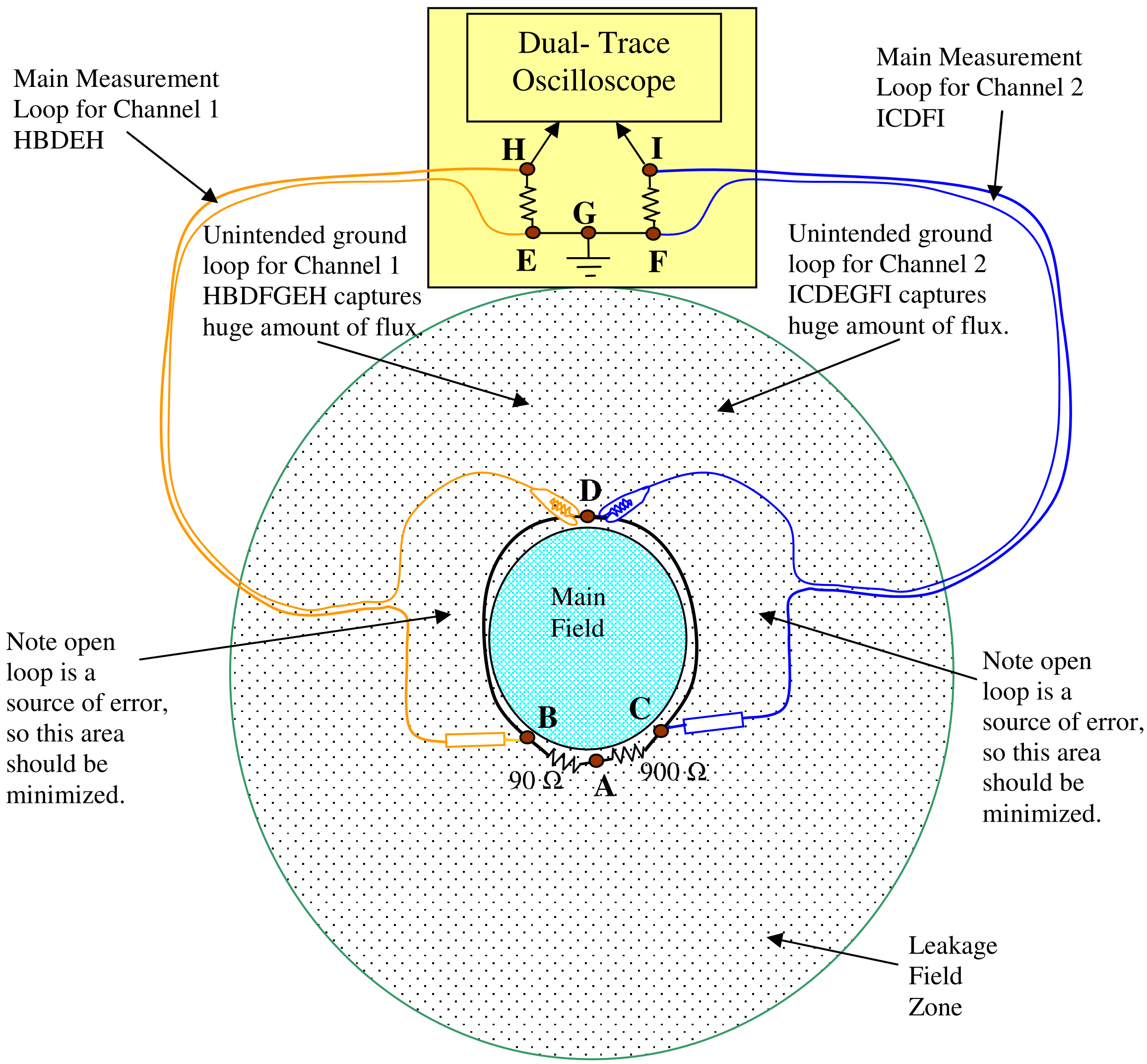

Fig. 1.4 Sources of error in WRONG! experimental setup

In order to improve the experimental setup and minimize the sources of error, the open loops shown in Fig. 1.4 should be closed up as much as possible. Fig. 1.5 shows an improved experimental setup in which this strategy is used. The issues of the unintended ground loops are minimized by allowing both scope probe cables to lay on top of each other, running in parallel. Further, it can be seen that the scope probe and ground connections are routed in a way that minimizes the area of the main measurement loop.

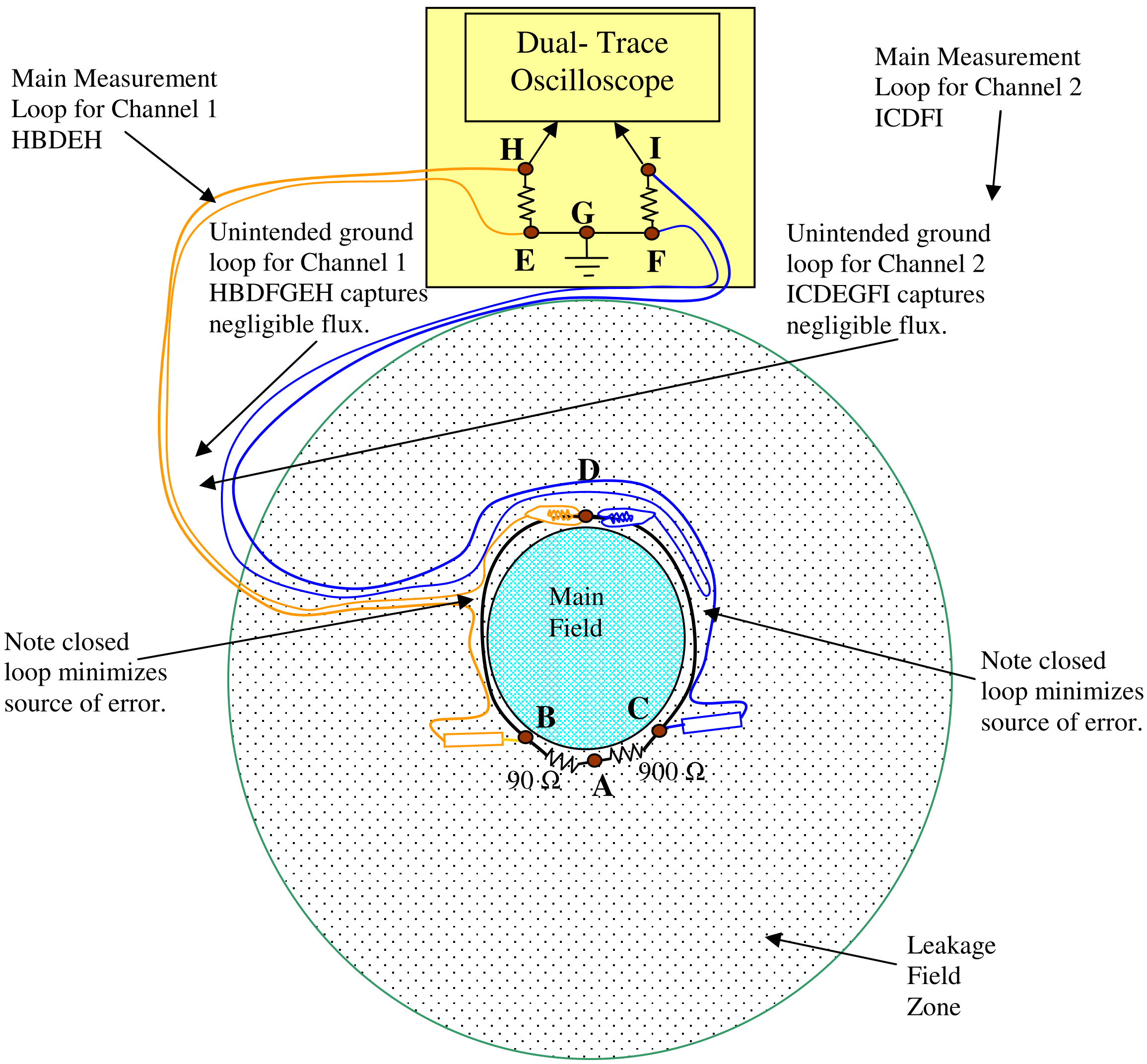

Fig. 1.5 Improved experimental setup

2. Main Flux-Generating Coil for Experiment

2.1 Main Flux-Coil Design Formulas

An estimation of the design for the main flux coil can be made using a simple formula for the single layer air-core cylindrical inductor. Fig. 2.1 shows a diagram for the single layer air-core inductor and indicates the coil radius *r*, coil width *w* and conductor diameter *d*. The use of an air-core circumvents any complications due to nonlinearity/saturation and core losses. In a precise experiment, uncertainties should be removed (when possible) and all variables should be quantified in magnitude, not just via ratios (if possible). The use of an air core, combined with actual magnetic field measurements will lend an extra degree of reliability to the results. Of course, an air core results in lower field magnitude, but this is no issue provided the measurement has the sensitivity to resolve the important quantities of interest.

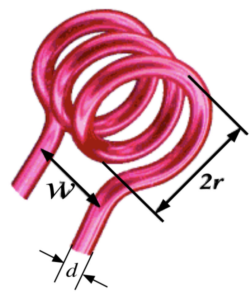

Fig. 2.1 Diagram of single-layer air-core cylindrical inductor

Eqn. 2.1 indicates the approximate inductance L_{coil} for this type of coil, assuming $w \gg r$, $d \ll w$ and assuming tight windings such that $w = N\,d$, where *N* is the number of turns. The coil resistance R_{coil} is given by eqn. 2.2 where σ is the conductivity of copper (~60x10^6 S/m). Equation 2.3 gives the flux Λ_{cen}, at the plane that cuts the coil in half, where I_{coil} is the current driving the coil. The on-axis magnetic field at the center of the coil B_{cen} is given by eqn. 2.4 and the on axis magnetic field at the end of the coil B_{end} is given by eqn. 2.5.

$$L_{coil} \approx (4\,\mu\text{H/m}) \cdot \frac{N^2\, r^2}{r + N\,d} \quad (2.1)$$

$$R_{coil} = \frac{8\,N\,r}{\sigma\, d^2} \quad (2.2)$$

$$\Lambda_{cen} = (4\,\mu\text{H/m}) \cdot \frac{N\,r^2 I_{coil}}{\sqrt{4r^2 + w^2}} \quad (2.3)$$

$$B_{cen} = (0.4\pi\,\mu\text{H/m}) \cdot \frac{N\,I_{coil}}{\sqrt{4r^2 + w^2}} \quad (2.4)$$

$$B_{end} = (0.4\pi\,\mu\text{H/m}) \cdot \frac{N\,I_{coil}}{2\sqrt{r^2 + w^2}} \quad (2.5)$$

2.2 Main Flux-Coil Design and Construction

A spool was chosen with approximate dimensions of r = 2.25 cm and w = 8.5 cm. The available wire was 18 ga. magnet wire with a diameter d = 0.109 cm. A tight single layer wrap of this wire on the spool resulted in a number of turns N = 78. Calculated results for this coil using the formulas in section 2.1 are as follows: L_{coil} =103 μH, R_{coil} =0.2 Ω, Λ_{cen} =(1.56 μH) I_{coil}, B_{cen} =(1.02 mT/A) I_{coil}, B_{coil} =(0.56 mT/A) I_{coil}

This is the basic design and Fig. 2.2 shows a picture of the completed coil after construction. This figure shows the coil being driven by a DC current of 1.105 A with a voltage of 0.301 V. While the meter readings on the power supply are not calibrated, it's clear that the actual resistance on the coil is closer to 0.27 Ω, hence the time constant (L/R) is estimated to be 0.38 ms. The discrepancy between measured and actual resistance is due to various uncertainties in the calculation, including: (i) approximate value of conductivity was used; (ii) length based on estimate from approximate coil radius and L=2πrN; (iii) lead lengths were not included in calculation; and (iv) that the wire thickness includes the insulating enamel coating, hence the diameter of the conducting copper portion of the wire is actually less than the value used.

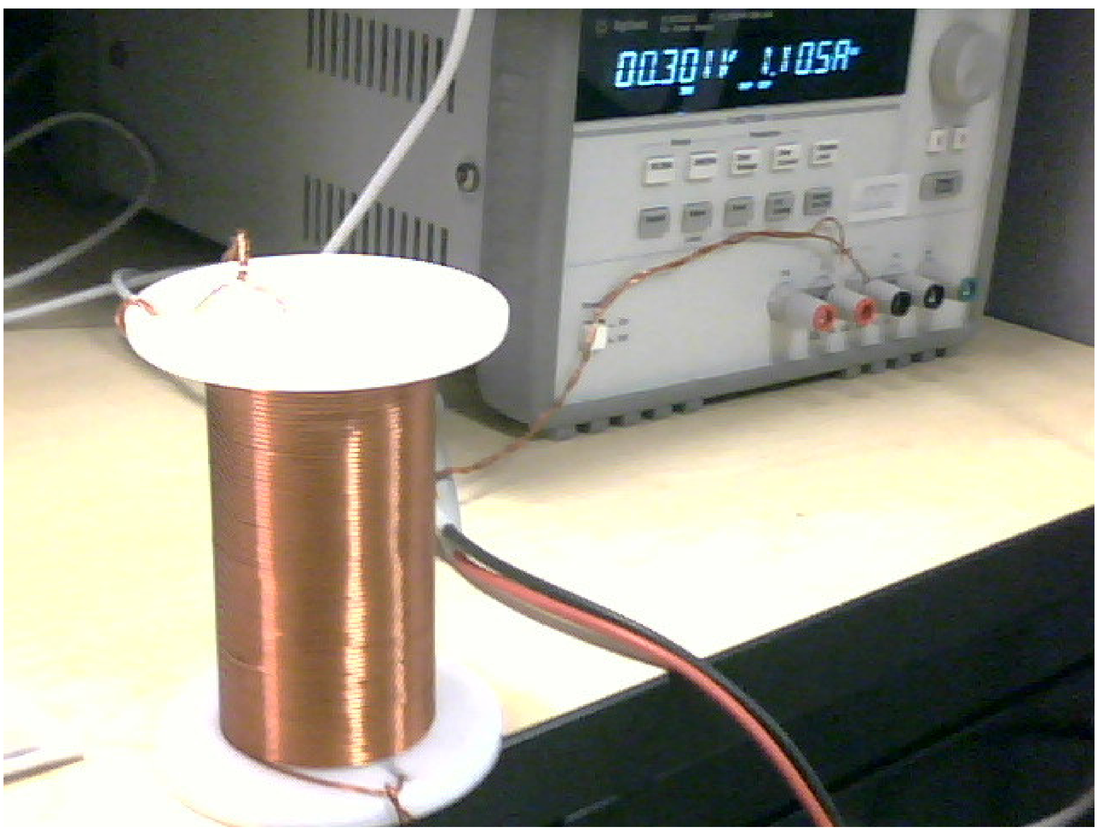

Fig. 2.2 Picture of completed coil

2.3 Experimental Verification of Coil Operation Under DC Conditions

The previous section showed the coil resistance was higher than calculated with a value near 0.27 Ohms. A more accurate reading was made with an Agilent 34401A milliohm meter shows the measured resistance is 0.287 Ohms. An inductance meter is not on hand and the actual inductance value is not critical for the measurement, however it is desirable to verify the magnetic performance of the coil in some way to verify that the theory and calculations are basically correct. A Hall-sensor based calibrated magnetic field meter was used to measure the coil magnetic field in the center B_{cen} and at the end B_{end} of the coil. The following table shows the results and makes a comparison of calculated and measured fields at various current values. The current range is 0 to 10 A, as expected in the final experiment. The table shows a slight dropping of the B_{cen} field from expected values as current increases, but this turned out to be due to temperature dependent offset drift on the Hall sensor, since the coil heated up considerably as 10 A was approached, and the probe was held in place with foam which preventing air flow from cooling. Once the coil cooled down, measurements returned to expected values. This measurement error could of course be calibrated out, but the measurements clearly indicate proper operation of the coil for the purposes of the intended experiment. The B_{end} measurements likely have discrepancy due to sensitivity of the fringe field, but there is reasonable agreement.

Table 1 Comparison of Calculated and Measured DC Magnetic Field Values

Current I_{coil} [A]	Center B_{cen} [mT]		Ends B_{end} [mT]	
measured	calculated	measured	calculated	measured
1.0	1.02	1.01	0.56	0.52
2.0	2.04	1.99	1.11	1.01
3.0	3.06	3.01	1.67	1.50
4.0	4.08	3.98	2.23	2.01
5.0	5.10	5.00	2.79	2.51
6.0	6.11	5.99	3.34	3.02
7.0	7.13	6.97	3.90	3.53
8.0	8.15	7.92	4.46	4.04
9.0	9.17	8.87	5.02	4.53
10.0	10.19	9.85	5.57	5.05

2.4 Experimental Measurement of Leakage Field

It is important to measure the approximate level of leakage field in the vicinity of the main drive coil because this field can penetrate any open measurements loops and corrupt the measurements. It should also be noted that even a small amount of leakage field can have a large impact as total flux because the main area of the center of the solenoid is relatively small (4.5 cm diameter circle), while the area of the entire outside experiment zone is 100 times greater (~50 cm diameter). Hence, a carelessly placed open loop generated by unintended ground paths can easily capture a total flux that may swamp out a sensitive measurement.

Fig. 2.3 shows a plot of the measured field as a function of radius from the solenoid. These measurements were made by driving the solenoid with a DC current of 3 A, and measuring with a calibrated Hall sensor based field meter. It can be noted that the field inside the solenoid is nearly constant in the center, but outside the solenoid the field decays. Still, field levels in the 1-10 percent range are clearly present in the critical measurement zone. Hence, it is clear that any experimental setup must keep open loops as small as possible to avoid capture of significant flux. It can be noted that the boundary condition at the coil radius is met to reasonable accuracy. The tangential magnetic field inside H_{t1} is about 2435 A/m, while the tangential magnetic field outside H_{t2} at the boundary is about –223 A/m. The difference is 2668 A/m which matches well with the current sheet density K_i of 2752 A/m (i.e. 3A, 78 turns, & length 8.5 cm).

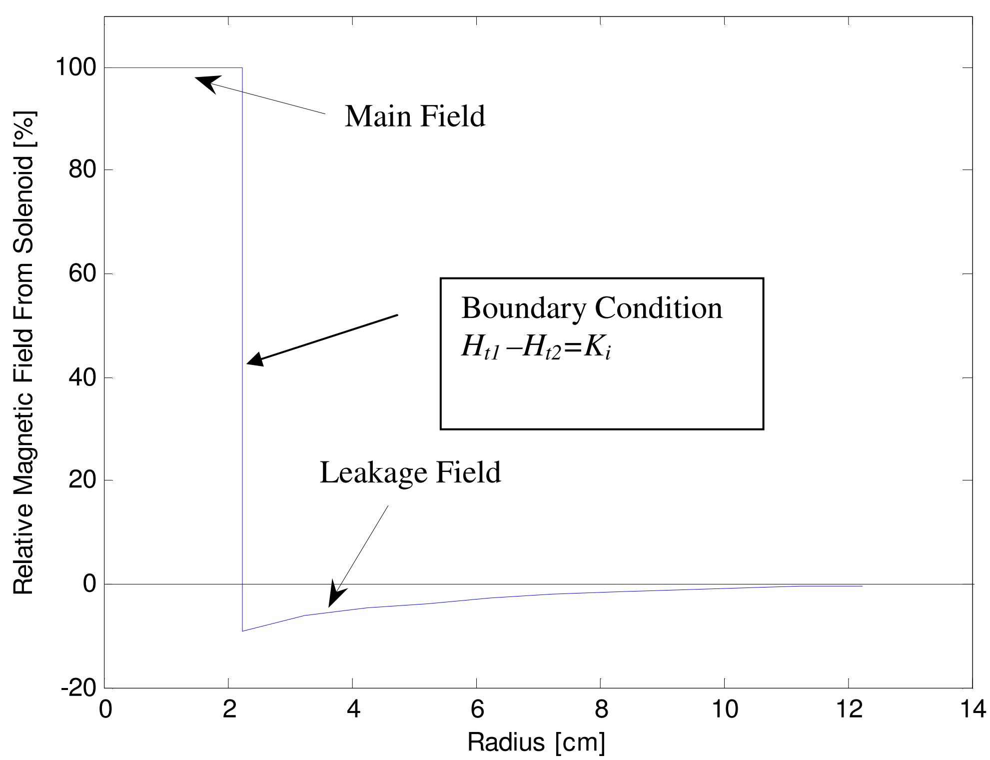

Fig. 2.3 Plot of relative magnetic field versus radius for main drive solenoid

2.5 Experiment Planning

Based on the given coil design, it's reasonable to plan for a current ramp from zero to 10 A, over a time period of 0.1 ms (i.e. $\frac{dI_{coil}}{dt} = 10^5$ A/s), which will allow 156 mV emf to be induced in a single loop that encircles the flux change. This voltage level is of the same order as other experiment and is sufficient to be easily measured with a scope. Note that any real coil has resistance; hence, it is not possible to use a step voltage to generate a continuous linear ramp. Instead an exponential shape will be found for the current. Still, the initial transient will include a nearly linear current ramp of sufficient duration for the oscilloscope to capture the measurement. The nice thing is that the maximum value of induced voltage will occur during this time, so a simple reading of the maximum voltage on the scope is able to reveal the data.

The coil itself will need to be driven by a voltage step of value as given in eqn. 2.6, which turns out to be 10.3 V. These numbers are such that an available Agilent E3633A 20 V, 10 A DC power supply with a switch to provide controlled step turn on capability will be suitable for the planned experiment.

$$V_{coil} = L_{coil} \frac{dI_{coil}}{dt} \tag{2.6}$$

Given that the time constant for the coil is 0.38 ms, there is no problem driving with a step voltage and obtaining brief period of a ramp current sufficient to capture with an O-scope. The power supply, with current limit protection, can be switched on, and the single transient captured by a digital scope. The supply current limit (10 A) will likely activate before 0.38 ms since a true current ramp (although it's really exponential) for 0.38 ms would produce 38 A. Still, these times scales are more than long enough for an O-scope to get the needed data.

The above analysis should be viewed as a guide to verify that the planned system can do the intended job. The analysis reveals that this is in fact true. **To obtain better measurements, a voltage step of 20 V will be chosen. This will then provide an estimated initial loop emf of 303 mV.**

3. Experimental Results

3.1 Verification of Prof. Lewin's Experiment

In the PF thread, there was a doubt placed on whether Prof. Lewin was actually connecting both O-scope probes to the same physical point as represented by nodes **A** and **D**. I personally did not see the claimed evidence in the video that the probes are not connected to the same point, and I have no reason to doubt the Prof. claim. Still, I can not verify the fact because the video is unclear (to my eyes) on this point. So, to remove any doubt, I simply redo Prof. Lewin's experiment and give pictures to show the connection points are the same. I then provide scope captures as proof that the results claimed by Prof. Lewin are correct.

Fig. 3.1a and Fig. 3.1b show the pictures of the physical experiment in line with that described in Fig. 1.2. Both scope channel grounds are tied at the same common point (node D) in back of the solenoid, and both scope probes are tied to the same common point (node A) in the front. Channel 1 monitors a 90 Ohm resistor as R_1 and channel 2 monitors a 900 Ohm resistor as R_2. In the image, the yellow trace (CH1) monitors the 90 ohm resistor, the blue trace (CH2) monitors the 900 ohm resistor and the red trace is the monitor for the applied voltage to the coil. The applied voltage is initially 20V, but then the power supply goes into current limit (set to 10A). All information on time scale and voltage scale for each trace is on the screen.

Fig. 3.1a Picture of Lewin experimental setup: front view

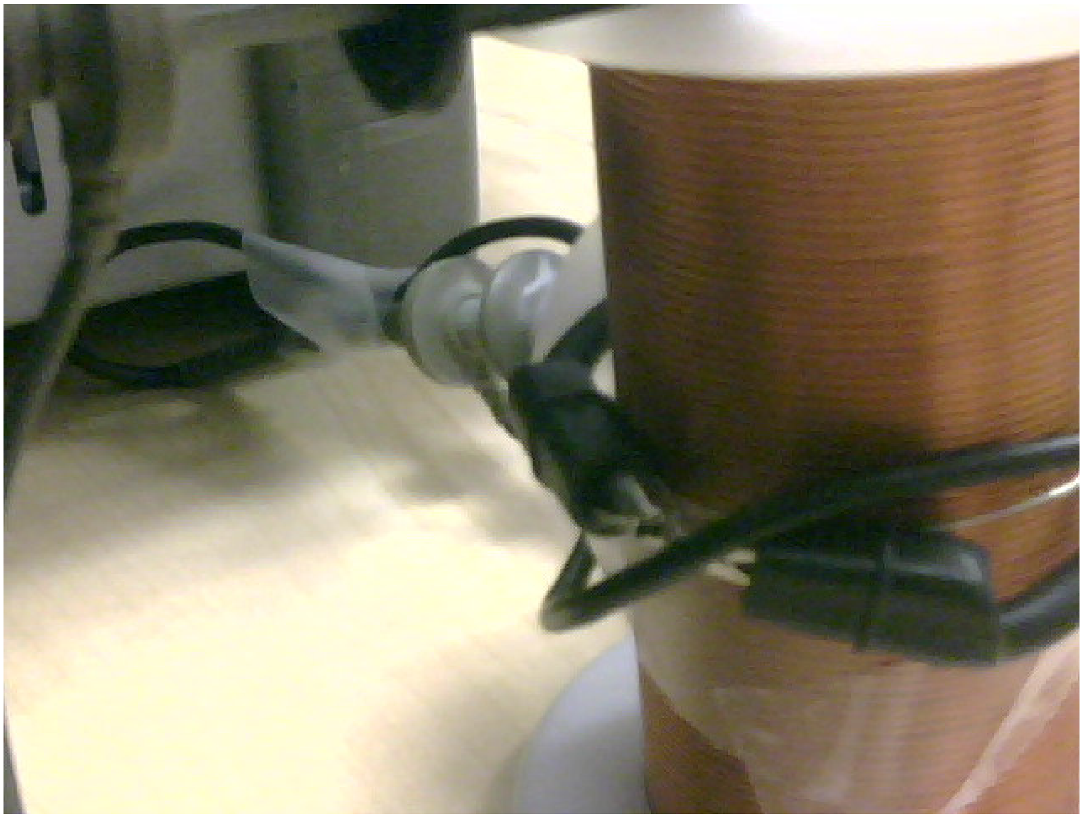

Fig. 3.1b Picture of Lewin experimental setup: back view

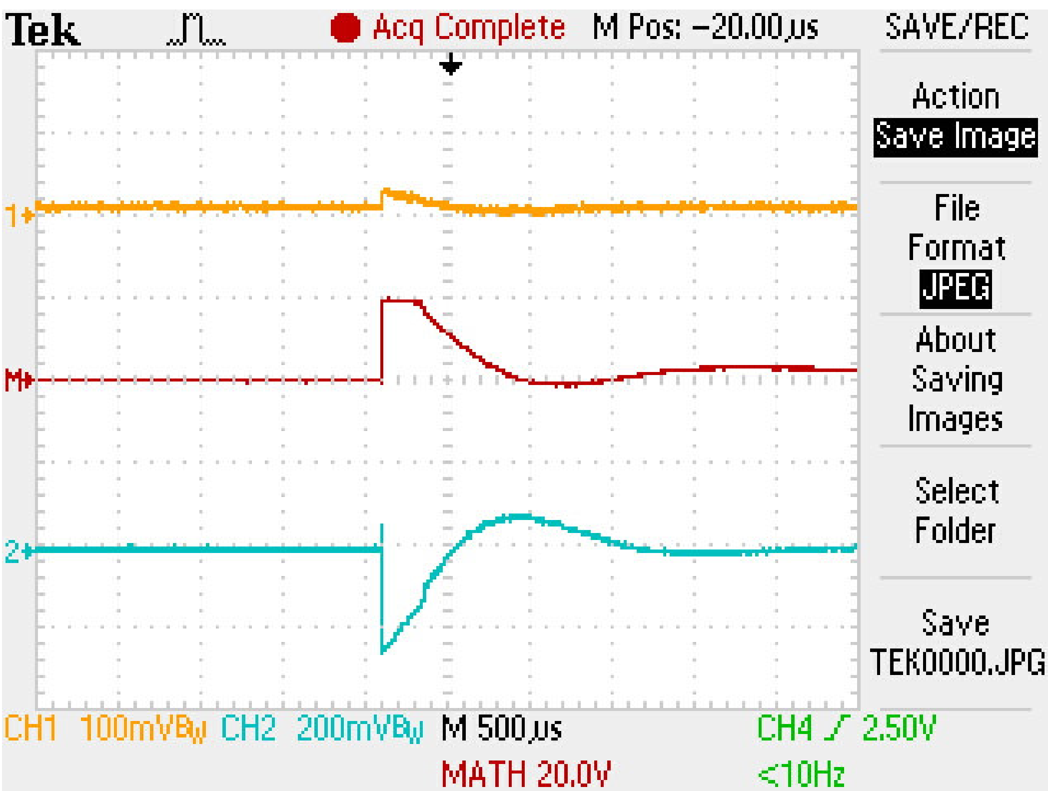

Fig. 3.2 Scope capture from Lewin experiment

3.2 Wire Measurement of Sarumonkey

In the PF thread, sarumonkey did a differential measurement on the wire itself and claimed to be able to measure the emf actually on the wire. My view is that he was instead just measuring emf induced by leakage flux in the very large loop created by the dual trace scope which has additional ground loops created by the common probe grounding in the scope.

Fig. 3.3a and Fig. 3.3b show the pictures of the physical experiment similar to that of sarumonkey. Both scope channel grounds are tied at the same common point (node D) in back of the solenoid. The difference here is that both scope probes are moved to the other side of their respective resistors (nodes B and C from previous discussions) in the front. Channel 1 monitors a 90 Ohm resistor as R_1 and channel 2 monitors a 900 Ohm resistor as R_2. Fig. 3.4 shows the results. In the image, the yellow trace (CH1) monitors the 90 ohm resistor, the blue trace (CH2) monitors the 900 ohm resistor and the red trace is the monitor for the applied voltage to the coil. It is clear that very little emf can be seen on either scope channel. The very small detected signals would seem to be just the little bit of flux that enters into the measurement loop. It is of course impossible to completely close up all loops, and all one can do is minimize leakage so that the generated emf is a small percentage of the total intended measurement.

Fig. 3.3a Picture of sarumonkey experimental setup: front view

Fig. 3.3b Picture of sarumonkey experimental setup: back view

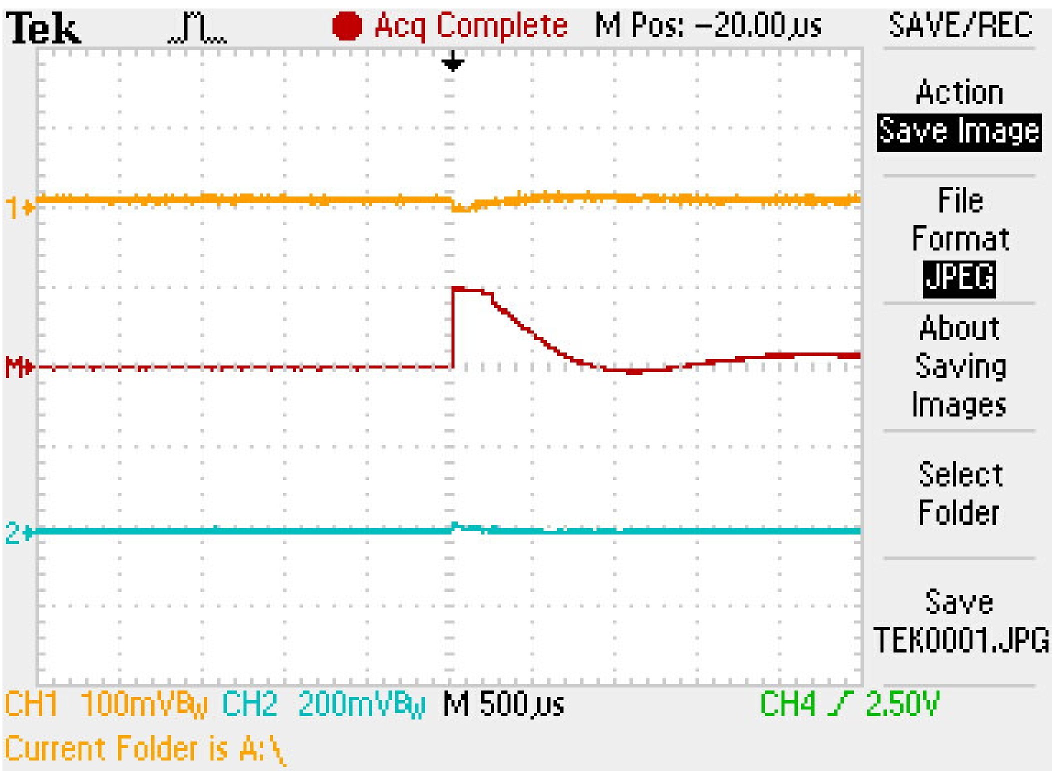

Fig. 3.4 Scope capture from sarumonkey experiment

4. Analysis of Experimental Results

This section provides an analysis of the experimental results of Section 3 to show reasonable agreement with Faraday's Law, under some assumptions. Fig. 4.1 shows a schematic diagram of the Lewin type of experiment using a dual trace oscilloscope functioning as the two voltmeters. The main flux for the main circuit loop is shown as Λ_M and the leakage fluxes are shown as Λ_1 , Λ_2, Λ_G and Λ_E. These leakage fluxes are inside measurement loop 1, inside measurement loop 2, inside the ground loops and external to the circuit, respectively. Note that the sum of the leakage fluxes must total the negative of the main circuit flux. The voltage measurements provided by both channels of the dual trace oscilloscope are V_1 for channel 1 and V_2 for channel 2. A full analysis requires consideration of all loops and all leakage fluxes, but a simplified analysis is possible under the assumption that Λ_1 , Λ_2, and Λ_G are negligible. The assumption that Λ_1 , Λ_2, and Λ_G are negligible, is confirmed to a reasonable degree in the measurements above for the sarumonkey arrangement. Here the leakage flux changes result in emf uncertainty at about the 20 mV level, which is less than 10 percent of the calculated main flux induced emf of 303 mV. The main measuremets of the Lewin experiment in Fig. 3.2 are $V_1 = -20$ mV and $V_2 = +280$ mV. These add to 300 mV when circling counterclockwise around the main circuit. This is well within experimental uncertainty and indicates a reasonably accurate experiment suitable to show concepts. This lends supports Prof. Lewin's expressed views.

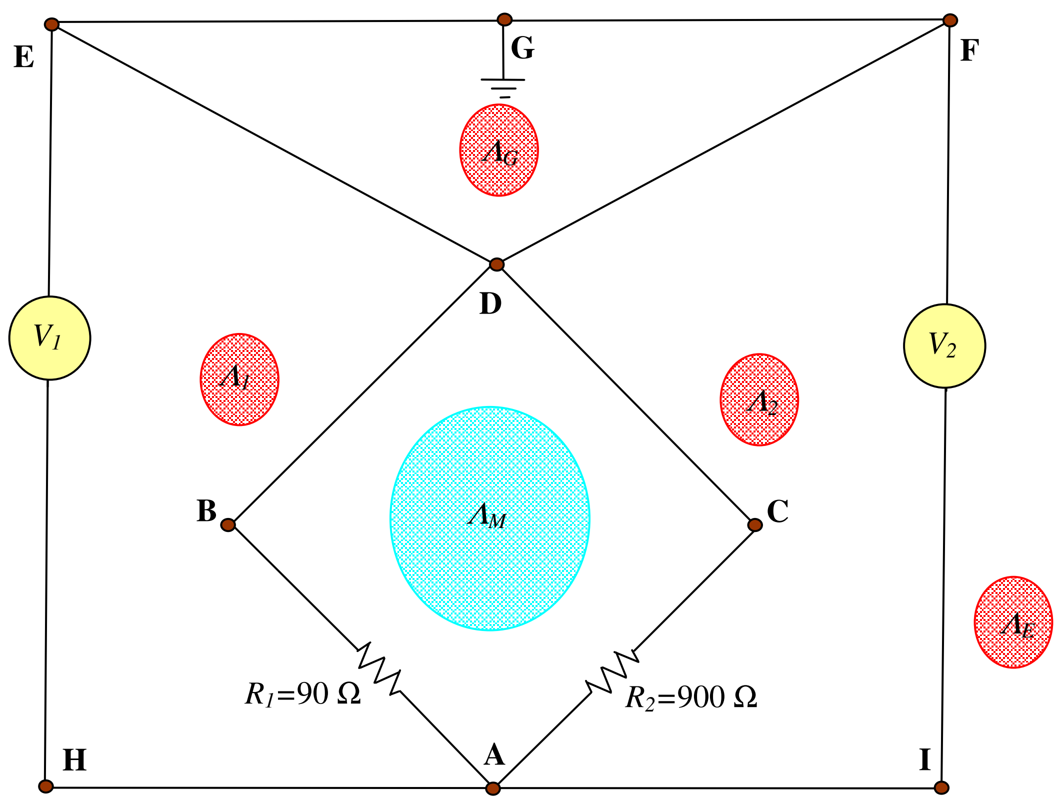

Fig. 4.1 Schematic representation of Lewin experimental using dual-trace O-scope