Problem Statement

Model a leaking pressurized container filled with an ideal gas (density ρ and molecular weight w) at a pressure P and temperature T. The rigid container has a fixed volume V and has a small leak (an adiabatic leak so that T doesn't change significantly) of area a. The leak rate will diminish with time since as material escapes, the driving pressure decreases. What function describes the pressure decay as a function of time, t?

Model

Start with ideal gas law:

$$P(t)V = RTn(t) \qquad (1)$$

Bernoulli flow equation: Equate stored energy density (aka pressure) P with kinetic energy density of the gas escaping from the leak hole, a:

$$P(t) = \frac{1}{2}\rho v^2(t) \qquad (2)$$

to obtain an expression for the speed of escaping gas:

$$v(t) = \sqrt{2P(t)/\rho} \qquad (3)$$

The molar leak rate S is proportional to the speed of the leaking gas

$$S(t) = \frac{a\rho v(t)}{w} = \frac{a}{w}\sqrt{2\rho P(t)} \qquad (4)$$

Inserting (1) into (4) yields

$$S(t) = \frac{a}{w}\sqrt{\frac{2\rho RT}{V}n(t)} \qquad (5)$$

or recognizing that the molar leak rate S is equal and opposite to the time derivative of the number of moles in the container, n:

$$\frac{dn}{dt} = -S(t) = -\frac{a}{w}\sqrt{\frac{2\rho RT}{V}}\sqrt{n(t)} \qquad (6)$$

Which, using (1), gives

$$\frac{dP}{dt} = -\frac{a}{w}\left(\frac{RT}{V}\right)^{3/2}\sqrt{2\rho}\sqrt{P(t)} = \text{constant} * \sqrt{P(t)} \qquad (7)$$