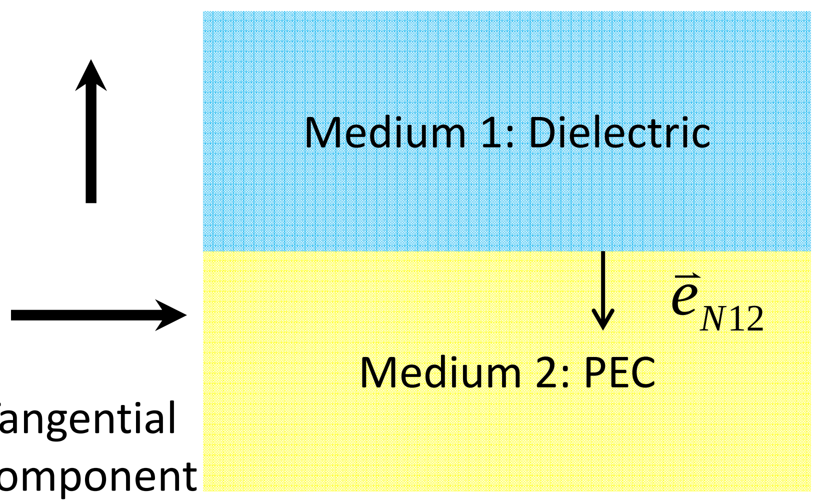
Reflection by PEC

Let us first consider PEC boundary conditions:

Normal Component



Tangential Component

$\vec{e}_{N12}$ Point from medium 1 to 2, along the normal direction

$$\left(\vec{H}_1 - \vec{H}_2\right) \times \vec{e}_{N12} = \vec{K}$$

$$\oint \vec{H} \cdot d\vec{L} = \frac{\partial}{\partial t} \iint \vec{D} \cdot d\vec{S} + \iint \vec{J} \cdot d\vec{S}$$

Within PEC: $\vec{D}_2 = 0$ $\vec{H}_2 = 0$

$\vec{E}_2 = 0$ $\vec{B}_2 = 0$

σ_s: surface charge on PEC boundary

$\vec{K}_s$: surface current on PEC boundary

Within Dielectric:

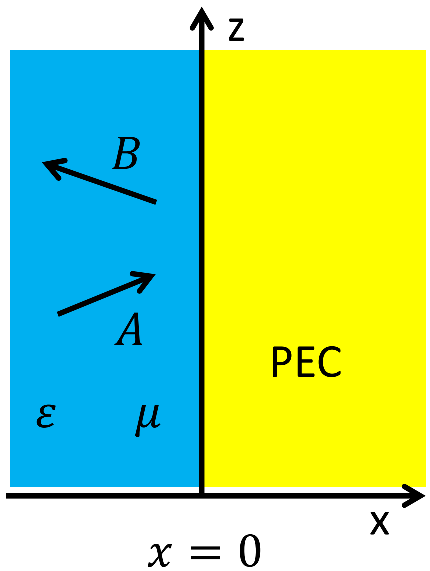
Normal Component $H_{N1} = 0$ $B_{N1} = 0$

$$|D_{N1}| = |\sigma_s| = \epsilon_1 |E_{N1}|$$

Tangential Component $E_{T1} = 0$ $D_{T1} = 0$

$$|H_{T1}| = |K_s| = \frac{1}{\mu_0} |B_{T1}|$$

Reflection by a PEC: TE Case



Incidence field:

$$\vec{E}_i = A\exp[j(\omega t - \beta z)]\exp[-jk_x x]\vec{e}_y$$

$$\vec{k} = \beta\vec{e}_z + k_x\vec{e}_x$$

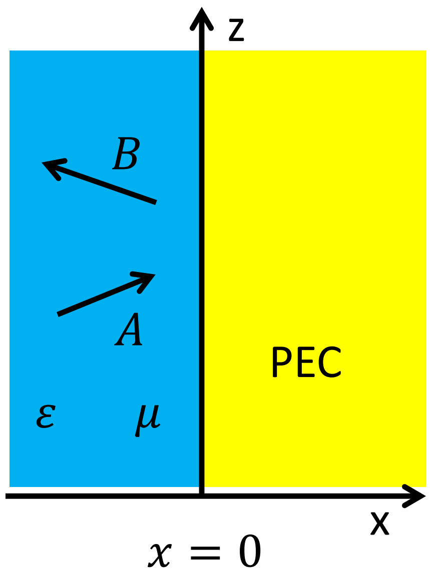
Total E field:

$$\vec{E} = \exp[j(\omega t - \beta z)]\{A\exp[-jk_x x] + B\exp[jk_x x]\}\vec{e}_y$$

From Maxwell's equations:

$$H_x = -\frac{j}{\omega\mu}\frac{\partial E_y}{\partial z} \qquad H_z = \frac{j}{\omega\mu}\frac{\partial E_y}{\partial x}$$

Reflection by a PEC: TE Case



$$H_x = -\frac{\beta}{\omega\mu}\{A\exp[-jk_x x] + B\exp[jk_x x]\}\exp[j(\omega t - \beta z)]$$

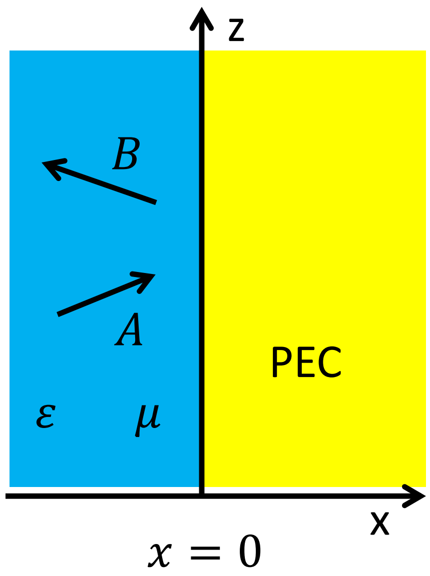
Boundary Condition: $H_x = 0$ @ x=0

$$B = -A$$

$$H_x = 2j\frac{\beta}{\omega\mu}A\sin(k_x x)\exp[j(\omega t - \beta z)] \qquad E_y = -2Aj\sin(k_x x)\exp[j(\omega t - \beta z)]$$

$$H_z = \frac{j}{\omega\mu}\frac{\partial E_y}{\partial x} \Rightarrow H_z = 2\frac{k_x}{\omega\mu}A\cos(k_x x)\exp[j(\omega t - \beta z)]$$

Reflection by a PEC: TE Case



$$E_y = -2Aj\sin(k_x x)\exp[j(\omega t - \beta z)]$$

$$H_x = 2j\frac{\beta}{\omega\mu}A\sin(k_x x)\exp[j(\omega t - \beta z)]$$

$$H_z = 2\frac{k_x}{\omega\mu}A\cos(k_x x)\exp[j(\omega t - \beta z)]$$

We can check that the following equation is satisfied

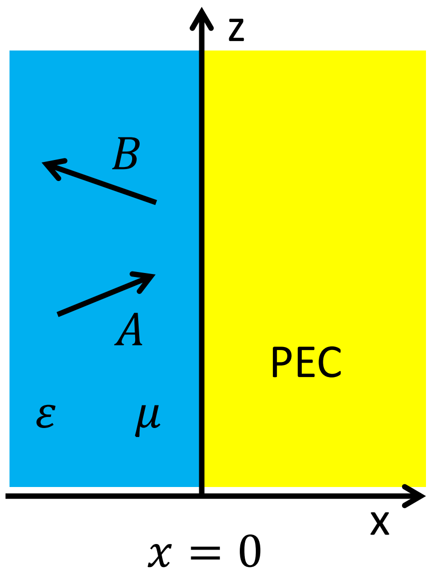
$$H_x = -\frac{j}{\omega\mu}\frac{\partial E_y}{\partial z}$$

Now let us consider what would happen at the boundary:

$$x = 0 \qquad E_y = 0 \qquad H_x = 0 \qquad H_z \neq 0$$

From E field value, we can know that there will be no surface charges.

Reflection by a PEC: TE Case



$$H_x(x=0)=0$$

$$H_z(x=0)=2\frac{k_x A}{\omega\mu}\exp[j(\omega t-\beta z)]$$

$$\vec{H}_1 = 2\frac{k_x A}{\omega\mu}\exp[j(\omega t-\beta z)]\vec{e}_z$$

$$\vec{H}_2 = 0$$

Surface Current Density

In PEC

$$(\vec{H}_1-\vec{H}_2)\times\vec{e}_{N12}=\vec{K}$$

In Dielectric

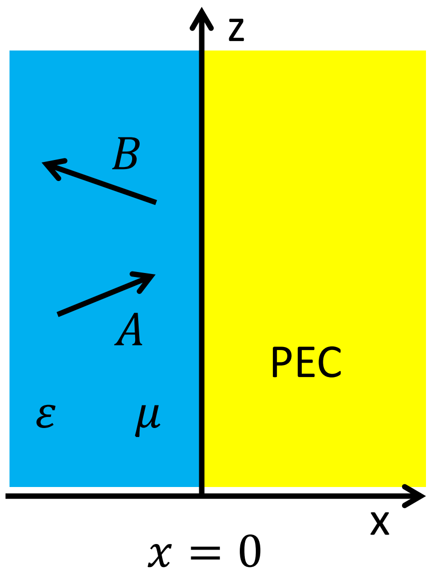
$$\vec{e}_{N12}=\vec{e}_x$$

$$\vec{K}=\vec{H}_1\times\vec{e}_x = 2\frac{k_x A}{\omega\mu}\exp[j(\omega t-\beta z)]\vec{e}_z$$

$$\vec{K}=2\frac{k_x A}{\omega\mu}\exp[j(\omega t-\beta z)]\vec{e}_z\times\vec{e}_x$$

$$\vec{K}=2\frac{k_x A}{\omega\mu}\exp[j(\omega t-\beta z)]\vec{e}_y$$

Reflection by a PEC: TE Case



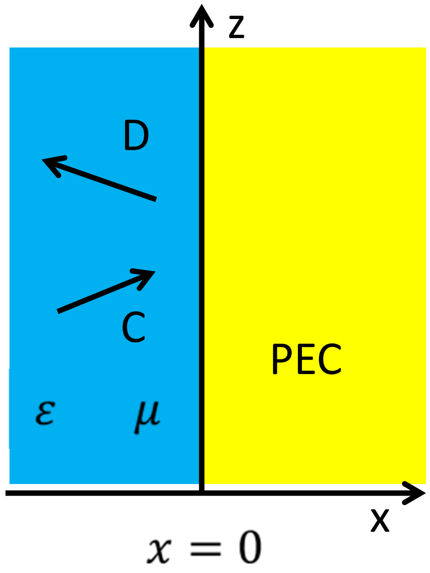
$$\vec{K} = 2\frac{k_x A}{\omega\mu}\exp\left[j(\omega t - \beta z)\right]\vec{e}_y$$

$$\vec{H}(x=0) = 2\frac{k_x A}{\omega\mu}\exp\left[j(\omega t - \beta z)\right]\vec{e}_z$$

We can also verify this result by verifying the following boundary condition:

$$\oint \vec{H}\cdot d\vec{L} = \frac{\partial}{\partial t}\iint \vec{D}\cdot d\vec{S} + \iint \vec{J}\cdot d\vec{S}$$

Reflection by a PEC: TM Case



$$\vec{H} = \exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] + D\exp[jk_x x]\}\vec{e}_y$$

$$\vec{E} = -\frac{j}{\omega\varepsilon}\nabla\times\vec{H}$$

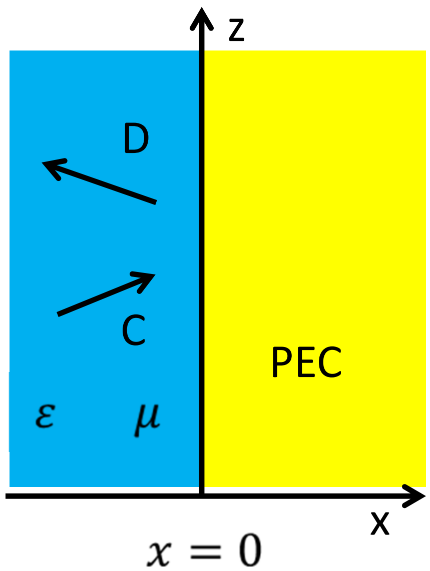
$$E_x = \frac{j}{\omega\varepsilon}\frac{\partial H_y}{\partial z}$$

$$E_z = -\frac{j}{\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$E_z = -\frac{k_x}{\omega\varepsilon}\exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] - D\exp[jk_x x]\}$$

$$E_x = \frac{\beta}{\omega\varepsilon}\exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] + D\exp[jk_x x]\}$$

Reflection by a PEC: TM Case



$$\vec{H} = \exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] + D\exp[jk_x x]\}\vec{e}_y$$

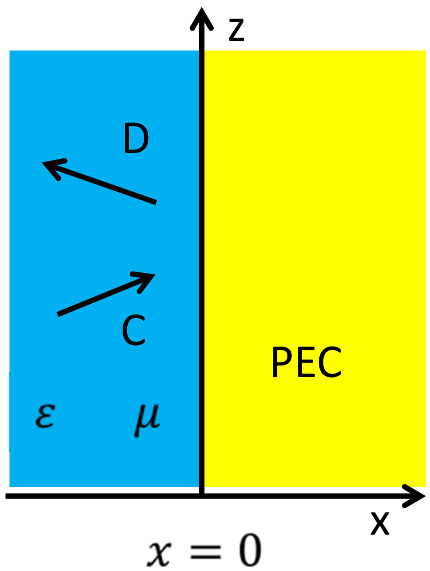
$$E_z = -\frac{k_x}{\omega\varepsilon}\exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] - D\exp[jk_x x]\}$$

$$E_x = \frac{\beta}{\omega\varepsilon}\exp[j(\omega t - \beta z)]\{C\exp[-jk_x x] + D\exp[jk_x x]\}$$

Boundary Condition: $E_z(x=0) = 0 \Rightarrow C = D$

$$\vec{H} = 2C\exp[j(\omega t - \beta z)]\cos(k_x x)\vec{e}_y$$

Reflection by a PEC: TM Case

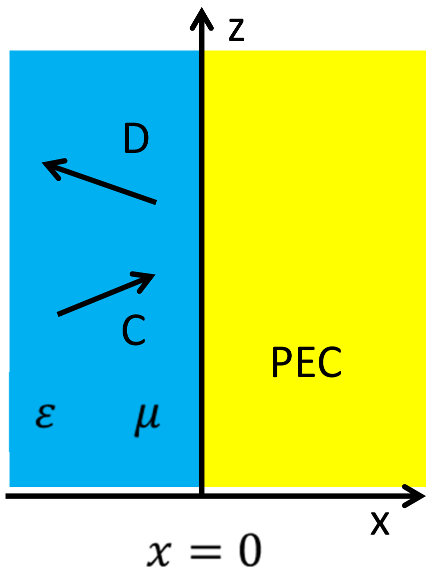


$$\vec{H} = 2C\exp\left[j\left(\omega t - \beta z\right)\right]\cos\left(k_x x\right)\vec{e}_y$$

$$E_x = 2\frac{\beta}{\omega\varepsilon}C\cos\left(k_x x\right)\exp\left[j\left(\omega t - \beta z\right)\right]$$

$$E_z = 2j\frac{k_x}{\omega\varepsilon}C\sin\left(k_x x\right)\exp\left[j\left(\omega t - \beta z\right)\right]$$

Reflection by a PEC: TM Case



Now let us consider what happens at the boundary

@ x=0

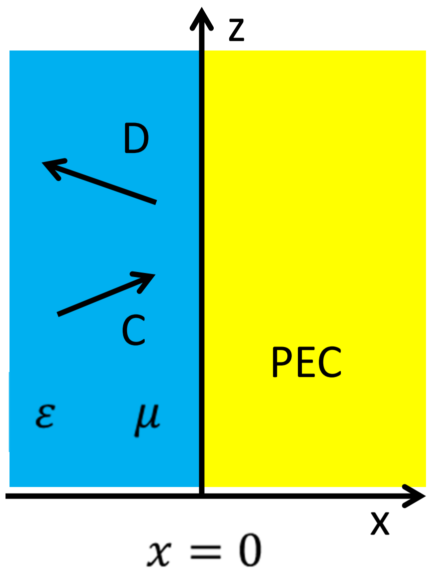
$$\vec{H}(x=0) = 2C\exp[j(\omega t - \beta z)]\vec{e}_y$$

$$\vec{E}(x=0) = 2\frac{\beta}{\omega\varepsilon}C\exp[j(\omega t - \beta z)]\vec{e}_x$$

Because of the E field boundary condition, there will be surface charge on PEC surface

$$\sigma = -2\frac{\beta}{\omega}C\exp[j(\omega t - \beta z)]$$

Reflection by a PEC: TM Case



Now let us consider what happens at the boundary

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$$\vec{H}(x=0) = 2C\exp[j(\omega t - \beta z)]\vec{e}_y$$

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Surface Current Density

In PEC

$$\left(\vec{H}_1 - \vec{H}_2\right)\times\vec{e}_{N12} = \vec{K}$$

In Dielectric

$$\vec{e}_{N12} = \vec{e}_x$$

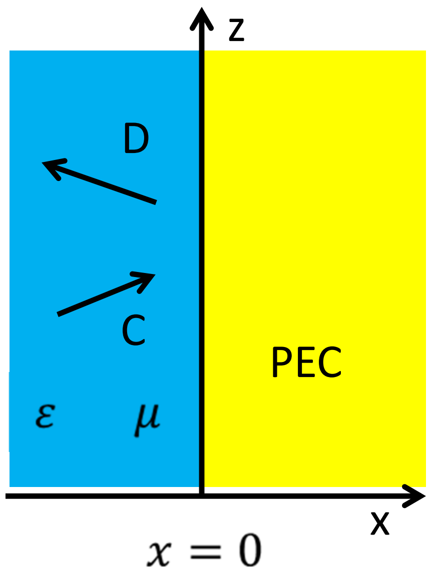
$$\vec{H}_1 = 2C\exp[j(\omega t - \beta z)]\vec{e}_y$$

$$\vec{H}_2 = 0$$

$$\vec{K} = 2C\exp[j(\omega t - \beta z)]\vec{e}_y \times \vec{e}_x$$

$$\vec{K} = -2C\exp[j(\omega t - \beta z)]\vec{e}_z$$

Reflection by a PEC: TM Case



$$\vec{H}_1 = 2C\exp\left[j\left(\omega t - \beta z\right)\right]\vec{e}_y$$

$$\vec{H}_2 = 0$$

$$\vec{K} = -2C\exp\left[j\left(\omega t - \beta z\right)\right]\vec{e}_z$$

We can again check that the integral form of the Maxwell's equations is satisfied:

$$\oint \vec{H} \cdot d\vec{L} = \frac{\partial}{\partial t}\iint \vec{D} \cdot d\vec{S} + \iint \vec{J} \cdot d\vec{S}$$