

Let  $P(x) = x^k + x^{k+1} + x^{k+2} + \dots + x^{k+n-1} + x^{k+n}$   
 So that  $P(x) = x^k(1 + x + x^2 + \dots + x^n)$

Let  $ml$  be equal to the number of terms in the parenthesis. Then we may factor  $P(x)$  as follows:

$$P(x) = x^k[(1 + x + x^2 + \dots + x^{m-1}) + x^m(1 + x + x^2 + \dots + x^{m-1}) + x^{2m}(1 + x + x^2 + \dots + x^{m-1}) + \dots + x^{(l-1)m}(1 + x + x^2 + \dots + x^{m-1})]$$

$$\text{So that } P(x) = x^k[(1 + x^m + x^{2m} + x^{3m} + \dots + x^{(l-1)m})(1 + x + x^2 + \dots + x^{m-1})]$$

This process may then be repeated for the factors obtained this way until the number of terms in each factor is prime.

As an example, we will use this technique to factor  $P(x) = 1 + x + x^2 + x^3 + \dots + x^{59}$ . This polynomial has 60 terms, so we will let  $m = 5$  and  $l = 12$ . We then have:

$$P(x) = (1 + x + x^2 + x^3 + x^4) + x^5(1 + x + x^2 + x^3 + x^4) + x^{10}(1 + x + x^2 + x^3 + x^4) + \dots + x^{50}(1 + x + x^2 + x^3 + x^4) + x^{55}(1 + x + x^2 + x^3 + x^4)$$

$$\text{Then } P(x) = (1 + x + x^2 + x^3 + x^4)(1 + x^5 + x^{10} + \dots + x^{50} + x^{55})$$

Observe that:

$$1 + x^5 + \dots + x^{55} = (1 + x^5 + x^{10}) + x^{15}(1 + x^5 + x^{10}) + x^{30}(1 + x^5 + x^{10}) + x^{45}(1 + x^5 + x^{10})$$

$$\text{So that } 1 + x^5 + \dots + x^{55} = (1 + x^5 + x^{10})(1 + x^{15} + x^{30} + x^{45})$$

$$\text{Furthermore, observe that } 1 + x^{15} + x^{30} + x^{45} = 1 + x^{15} + x^{30}(1 + x^{15}) = (1 + x^{15})(1 + x^{30})$$

$$\text{So we finally have } 1 + x + x^2 + x^3 + \dots + x^{59} = (1 + x + x^2 + x^3 + x^4)(1 + x^5 + x^{10})(1 + x^{15})(1 + x^{30})$$