Your Chap 5.2 cont. eqn example

Assume 2D and smooth ρ, u, v

Consider first term in (5.1), use Taylor exp.

$$\rho u(x,y)dy + (\rho u)_y(x,y)\frac{dy^2}{2} + (\rho u)_{yy}(x, y+\theta_3)\frac{dy^3}{6}$$

$$-(\rho u(x+dx,y)dy + (\rho u)_y(x+dx,y)\frac{dy^2}{2} + (\rho u)_{yy}(x, y+\theta_4)\frac{dy^3}{6}$$

$$(*) \quad = (\rho u)_x(x+\theta_1, y)dxdy + (\rho u)_{xy}(x+\theta_2, y)\frac{dxdy^2}{2} + \text{—''—}$$

with $0 < \theta_j < dy = dx$. Divide with $dxdy = dy^2$

Do same with remaining terms of (5.1)

Standard math: Take limit $dx = dy \to 0 \Rightarrow$ (5.3)

Nonstandard math: By the transfer property

$\frac{(*)}{dxdy}$ can be considered for $dx = dy$ infinitesimal

cont. $\Rightarrow (\rho u)_x(x+\theta_1, y) \approx (\rho u)_x(x,y)$

$(\rho u)_{xy}(x+\theta_2, y)\,dy \approx 0$, $(\rho u)_{yy}(x, y+\theta_3)dy \approx 0$

$(\rho u)_{yy}(x+dx, y+\theta_4)dy \approx 0$

Analogously for other terms of (5.1) $\Rightarrow$

(5.3) holds modulo infinitesimal $\Big\} \Rightarrow$ (5.3) holds.

(5.3) is standard