a) $V = \mathbb{R} \geq 1$; $x \oplus y = \max(x,y)$, $z = 1$

R1

$$x \oplus y = \max(x,y)$$

$$y \oplus x = \max(y,x) = \max(x,y)$$ (V)

LHS = RHS

R2

$$(x \oplus y) + z \overset{?}{=} x + (y \oplus z)$$

$$\max(x,y) + z \overset{?}{=} x + \max(y,z)$$

$$\max(x,y) + 1 \overset{?}{=} x + y$$ (X)

Choose:

$x = 5$

$y = 3$

LHS : $\max(5,3) + 1 = 6$

RHS $= 5 + 3 = 8$

R3

a) $z \oplus x = x \Rightarrow \max(z,x) = x$

b)