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assumptions made

Along OA, $y = 0$, $x = t$, $t : 0 \to 3$, $dx = dt$, $dy = 0$.

Along AB, $x = 3$, $y = t$, $t : 0 \to 2$, $dx = 0$, $dy = dt$.

Along BO, $x = t$, $y = \frac{2}{3}t$, $t : 3 \to 0$, $dx = dt$, $dy = \frac{2}{3}\,dt$.

$$\text{Hence,}\quad \oint (2x - y + 4)\,dx + (5y + 3x - 6)\,dy$$
$$= \int_0^3 (2t + 4)\,dt + \int_0^2 (5t + 3)\,dt + \int_3^0 \frac{50t}{9}\,dt = 12.$$

4. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$ and $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$; $t : 0 \to 1$.

Solution:

$$\begin{aligned}\int_C \mathbf{F} \cdot d\mathbf{r} &= \int_C z\,dx + x\,dy + y\,dz \\ &= \int_0^1 (t^3 + t.2t + t^2.3t^2)\,dt \\ &= \left[\frac{t^4}{4} + \frac{2t^3}{3} + \frac{3t^5}{5}\right]_0^1 = \frac{91}{60}.\end{aligned}$$

5. Calculate the work done by the force field $\mathbf{F} = 3xy\,\mathbf{i} - 2\,\mathbf{j}$ in moving from $A : (1, 0, 0)$ to $B : (2, \sqrt{3}, 0)$

(a) along the straight line AB;

(b) along the straight lines from A to $D : (2, 0, 0)$ and then from D to B;

(c) along the piece of the hyperbola $x^2 - y^2 = 1$, $z = 0$ from A to B.

Solution: The work done by $\mathbf{F}$ along a path C is

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C 3xy\,dx - 2\,dy + 0\,dz.$$

$$\frac{d\mathbf{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

(a) The straight line joining AB has equation $\mathbf{r} = \mathbf{i} + t(\mathbf{i} + \sqrt{3}\mathbf{j})$ or $x = 1 + t$, $y = \sqrt{3}t$, $z = 0$; $\quad t : 0 \to 1$. Thus

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 (3(1+t)\sqrt{3}t\,dt - 2\sqrt{3}\,dt) = \sqrt{3}/2.$$

(b)

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{AD} \mathbf{F} \cdot d\mathbf{r} + \int_{DB} \mathbf{F} \cdot d\mathbf{r}.$$

On AD, $y = 0$, $dy = 0$ so

$$\int_{AD} \mathbf{F} \cdot d\mathbf{r} = 0$$

On DB, $x = 2$, $dx = 0$ so

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$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_{DB} \mathbf{F} \cdot d\mathbf{r} = \int_0^{\sqrt{3}} -2\,dy = -2\sqrt{3}.$$