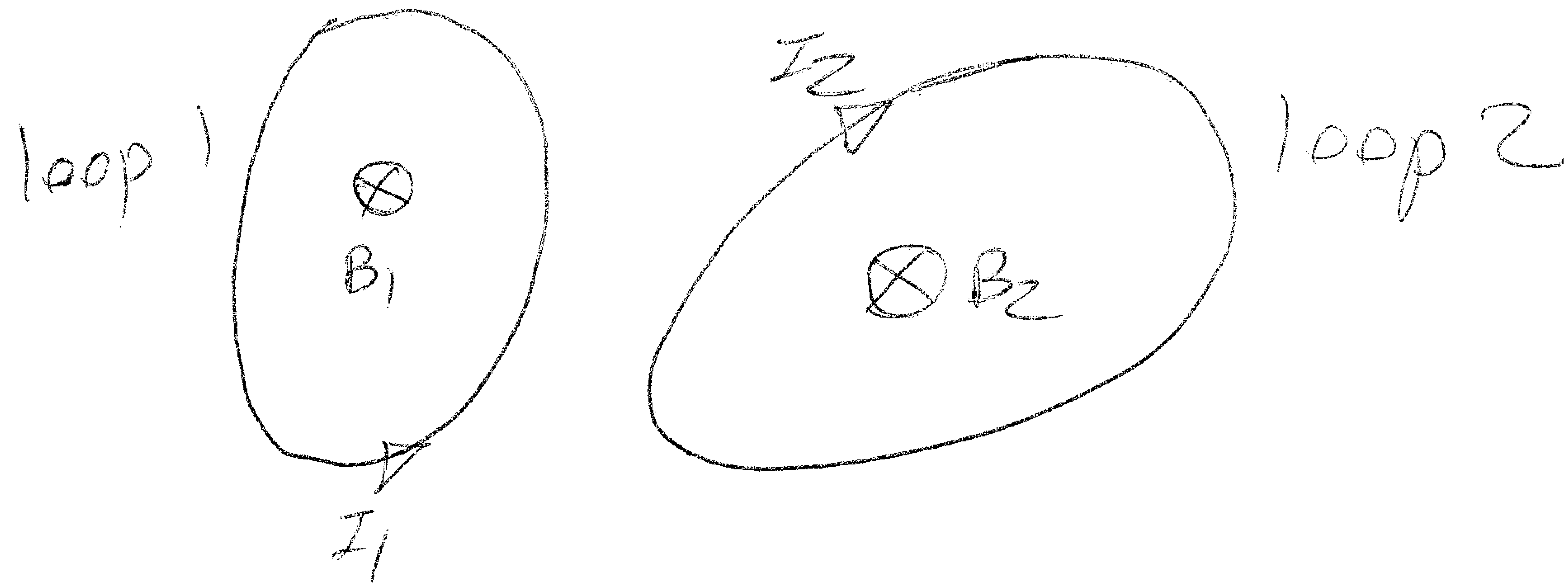
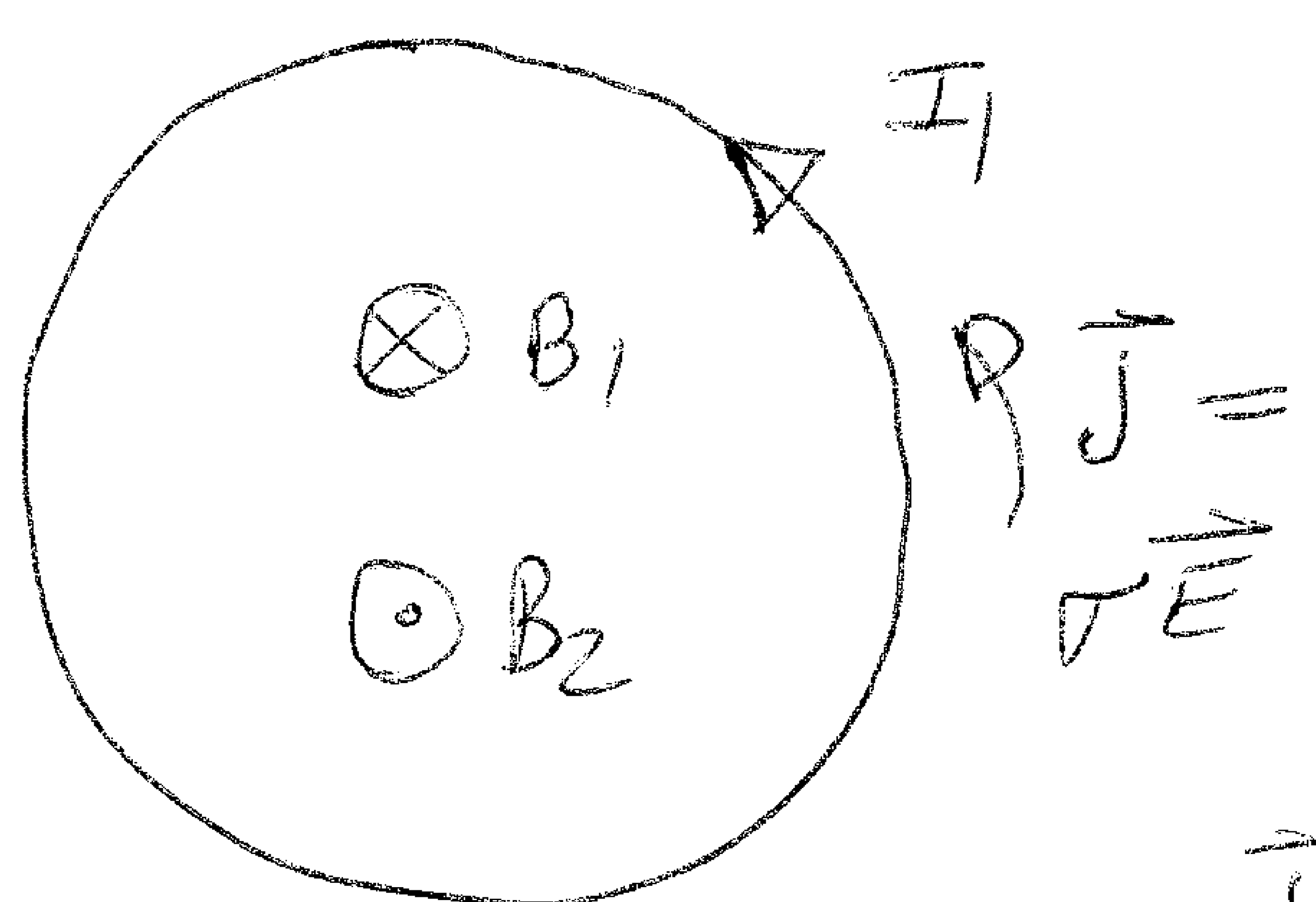
Claude Abraham 29 July 2012

Torque between 2 current-carrying loops



Let us isolate loop 1 & examine the force/torque due to loop 2.



Ohm's Law –
$\vec{J} = \sigma\vec{E}$, &
work done is related to $\vec{J}\cdot\vec{E}$

$$\vec{J}\cdot\vec{E} = (\sigma\vec{E})\cdot\vec{E} = \sigma|\vec{E}|^2$$

Since $\vec{\nabla}\cdot\vec{B} = 0$, &

$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$, for any $\vec{A}$, we have $\vec{B} = \vec{\nabla}\times\vec{A}$

$\vec{A}$ is the magnetic vector potential.

Maxwell equation from Faraday Law: $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$

but $\vec{B} = \vec{\nabla}\times\vec{A}$, so that $\vec{\nabla}\times\vec{E} = -\dfrac{\partial}{\partial t}(\vec{\nabla}\times\vec{A})$

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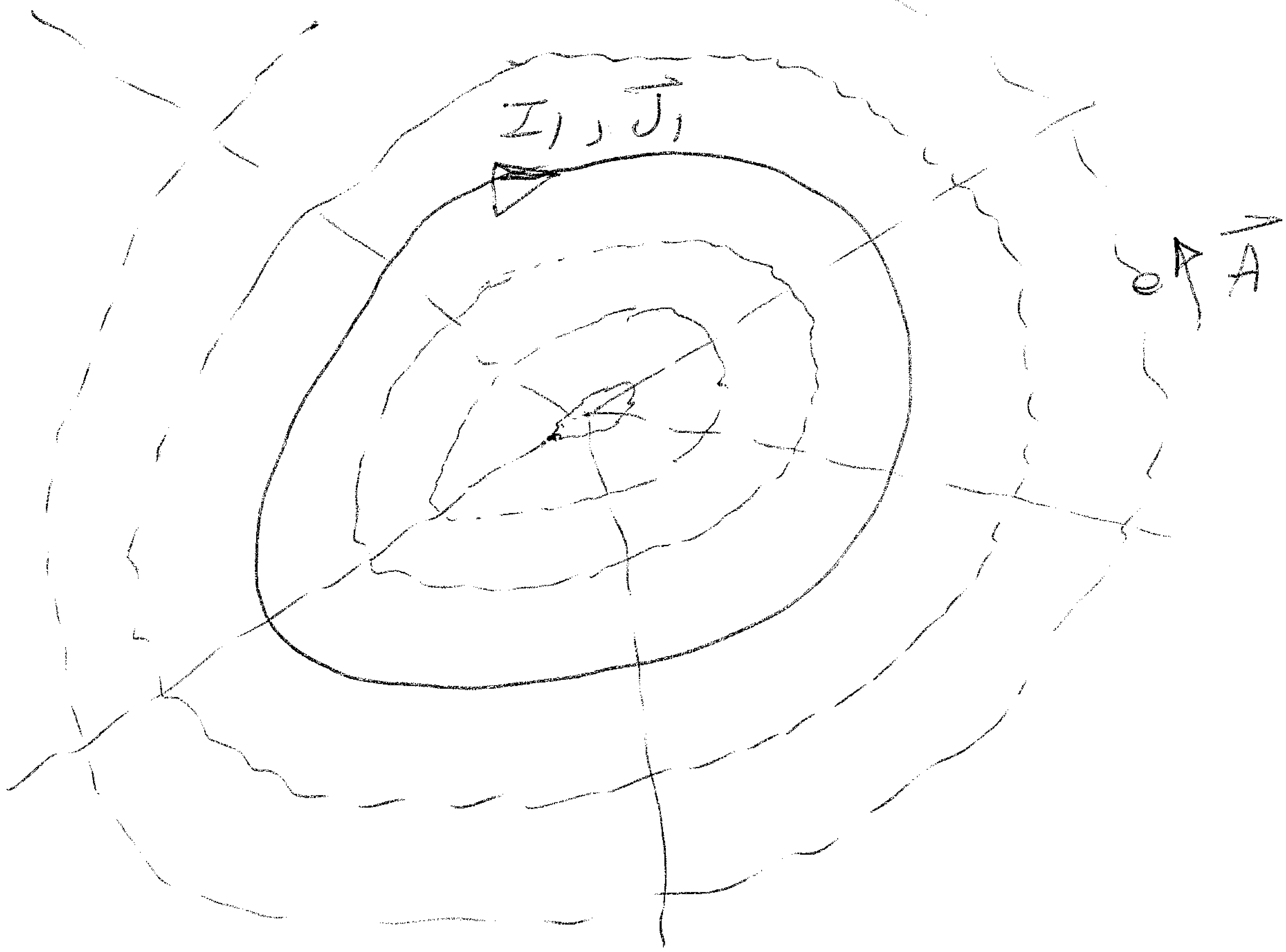
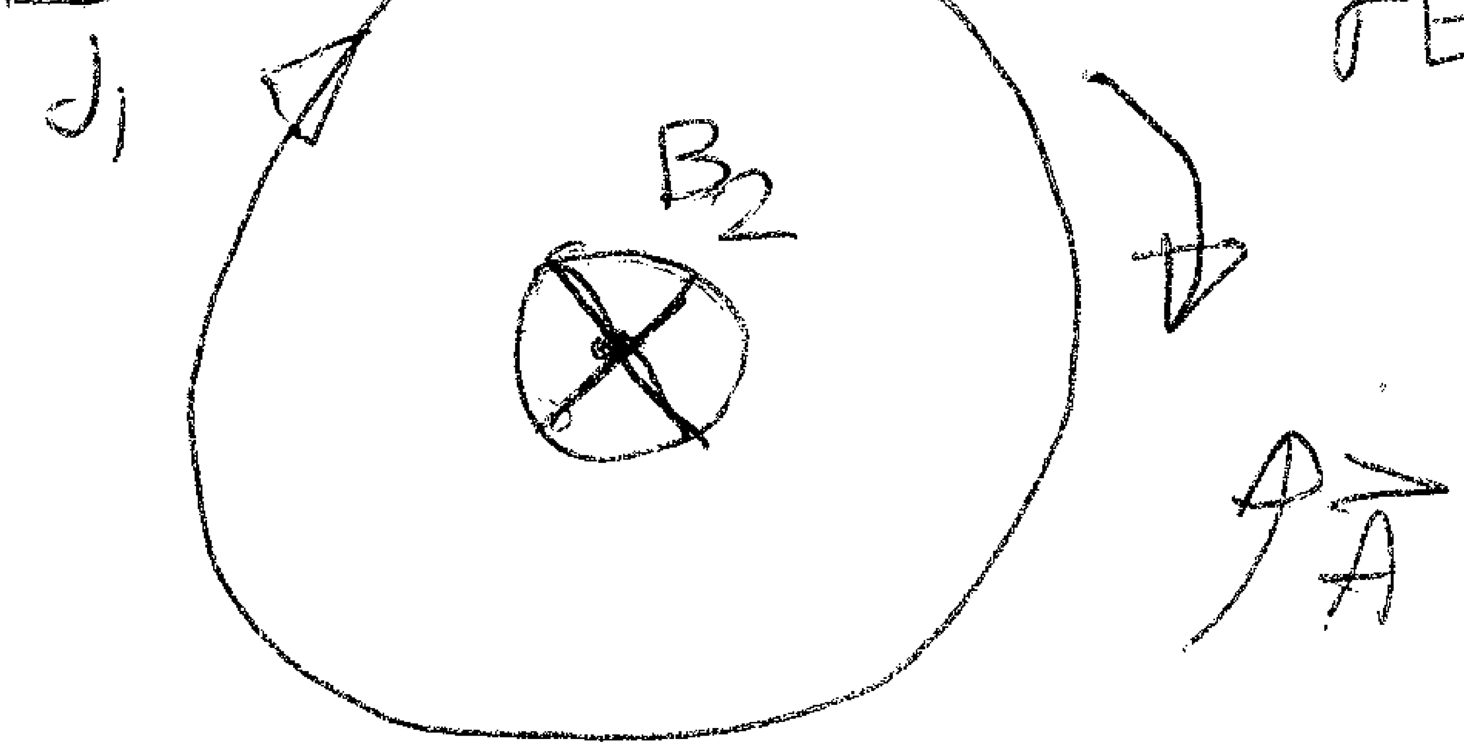
$\therefore \vec{E} = -\dfrac{\partial \vec{A}}{\partial t}$, where $\vec{E}$ is a non-conservative (rotational) type of field.

The vector $\vec{A}$ runs parallel to $\vec{J}$ as follows

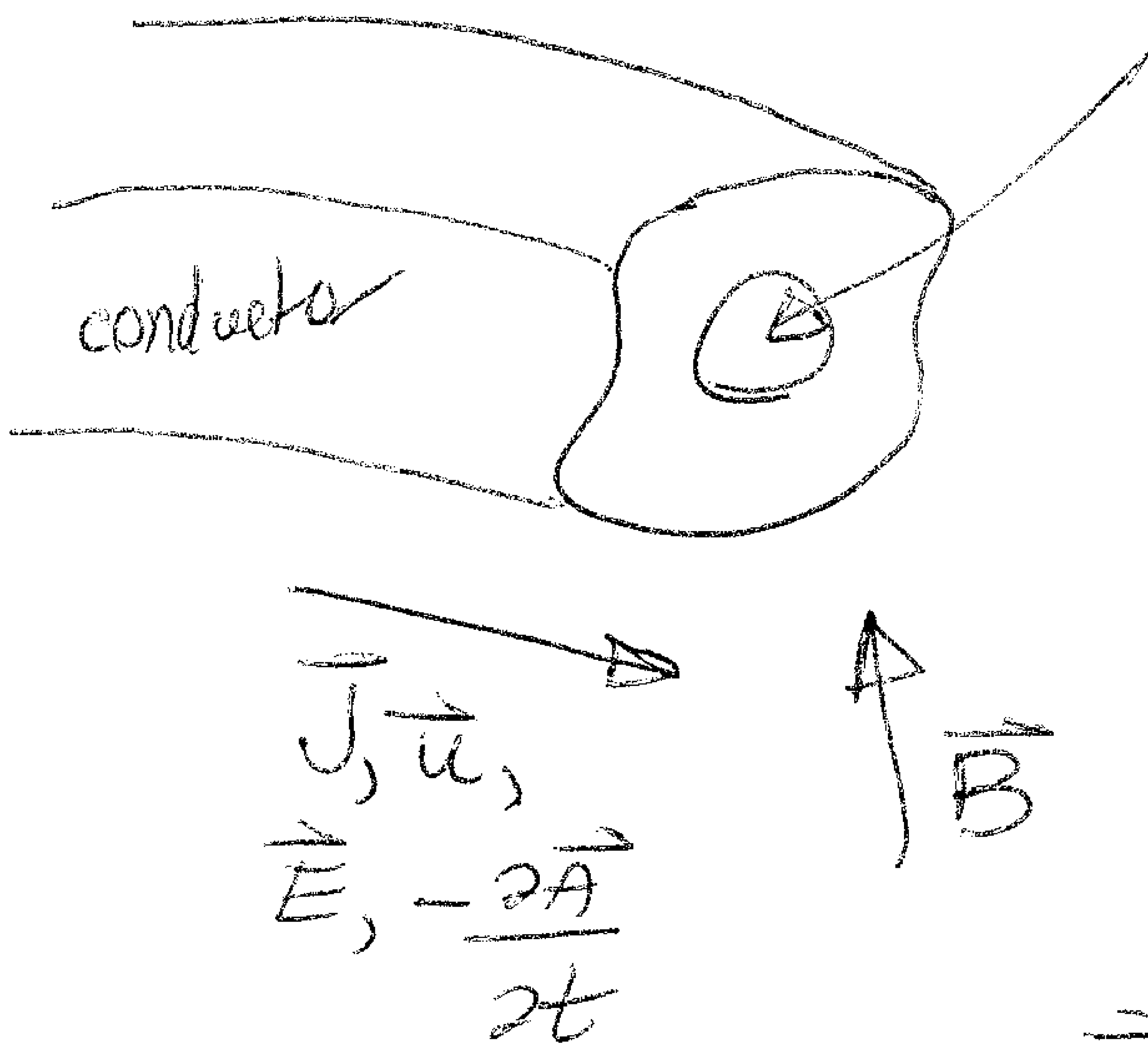


The closed loops inside and outside the loop current J_1, are the constant magnitude values of $\vec{A}$, the radial lines extending from the center are constant angle



$$\sigma \vec{E}_1 = \sigma\left(-\frac{\partial \vec{A}}{\partial t}\right)$$

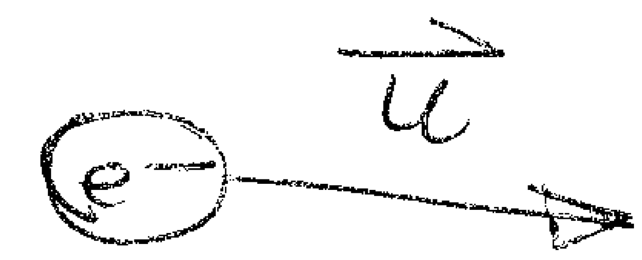
Let us now examine Lorentz force in loop 1.



electron, $\vec{u}$ = electron velocity

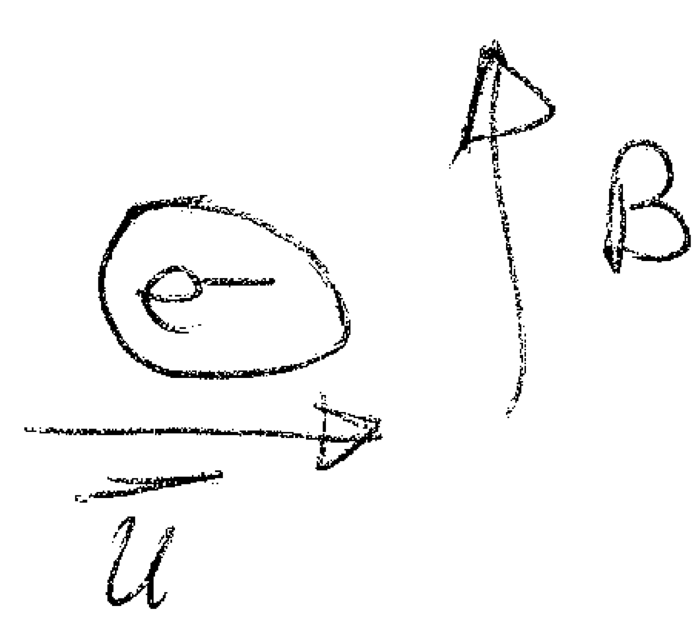
The force on electron is $\vec{F} = q(\vec{E} + \vec{u} \times \vec{B})$

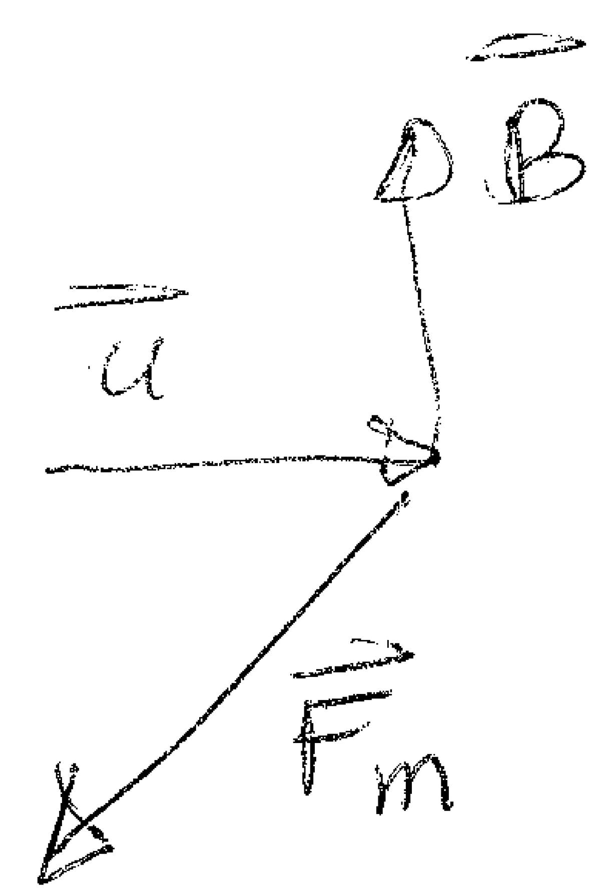
The electric force F_e is $q\vec{E}$



$\vec{E}$ acts in the direction of electron velocity $\vec{u}$, so that $\vec{E}$ provides work on electron to provide conduction & restore energy lost due to resistive collisions. The magnetic force is given by:

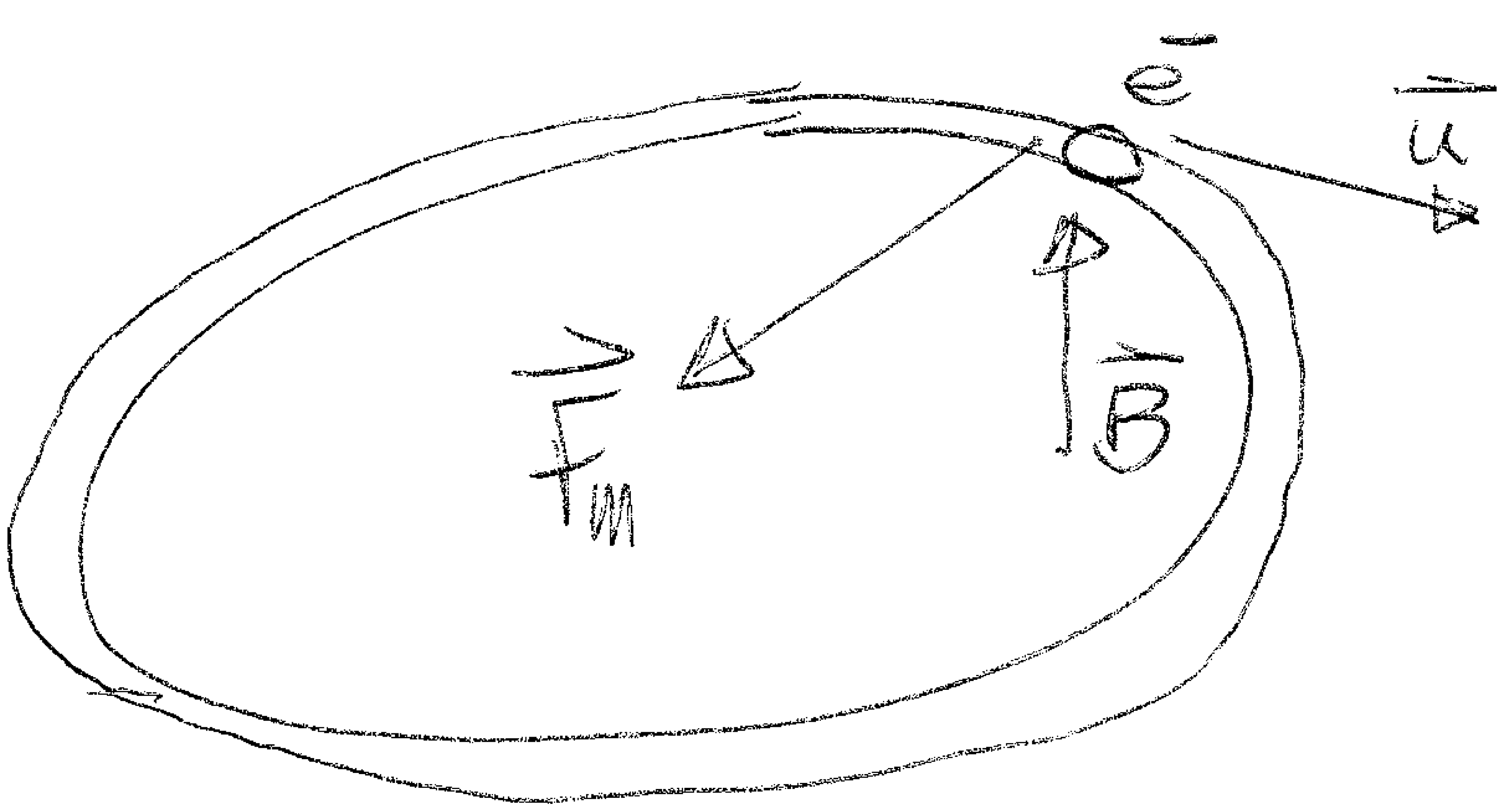
$$\vec{F}_m = q(\vec{u} \times \vec{B})$$



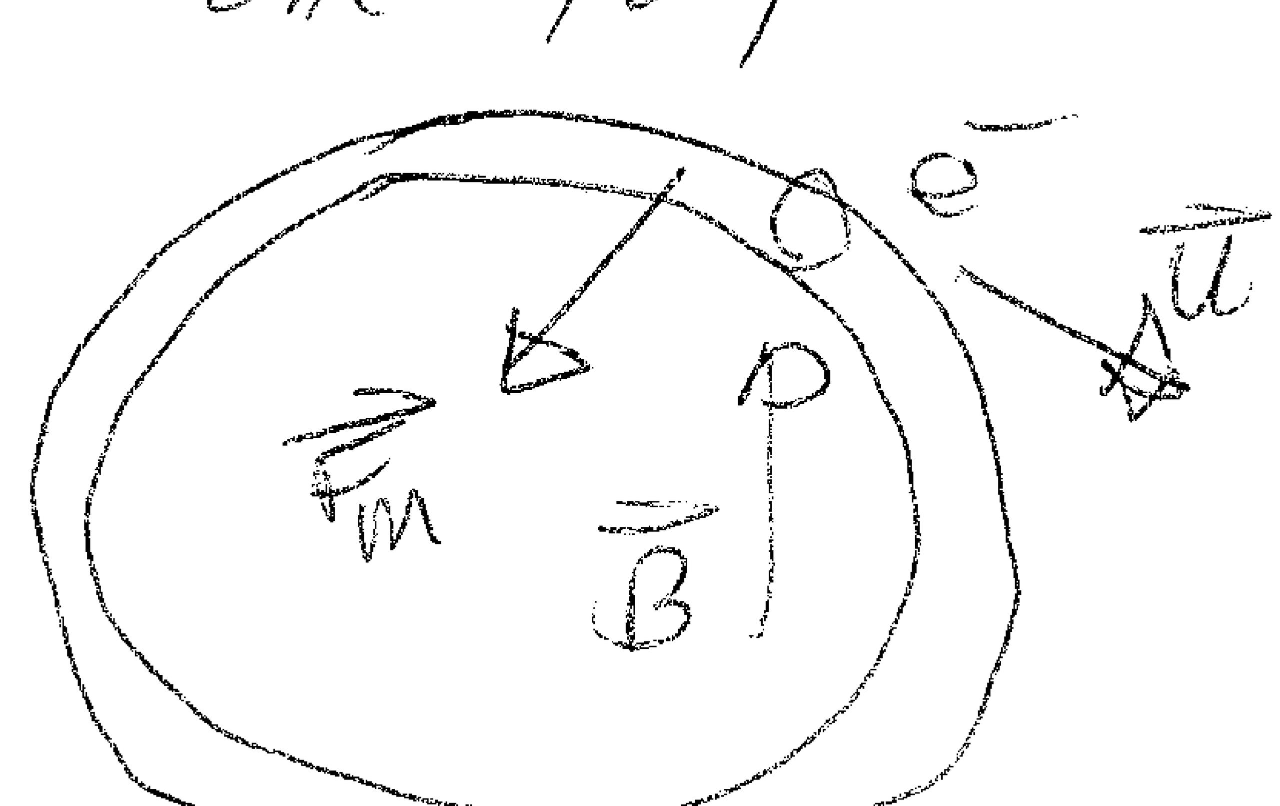


The cross product is normal to $\vec{u}$ & $\vec{B}$

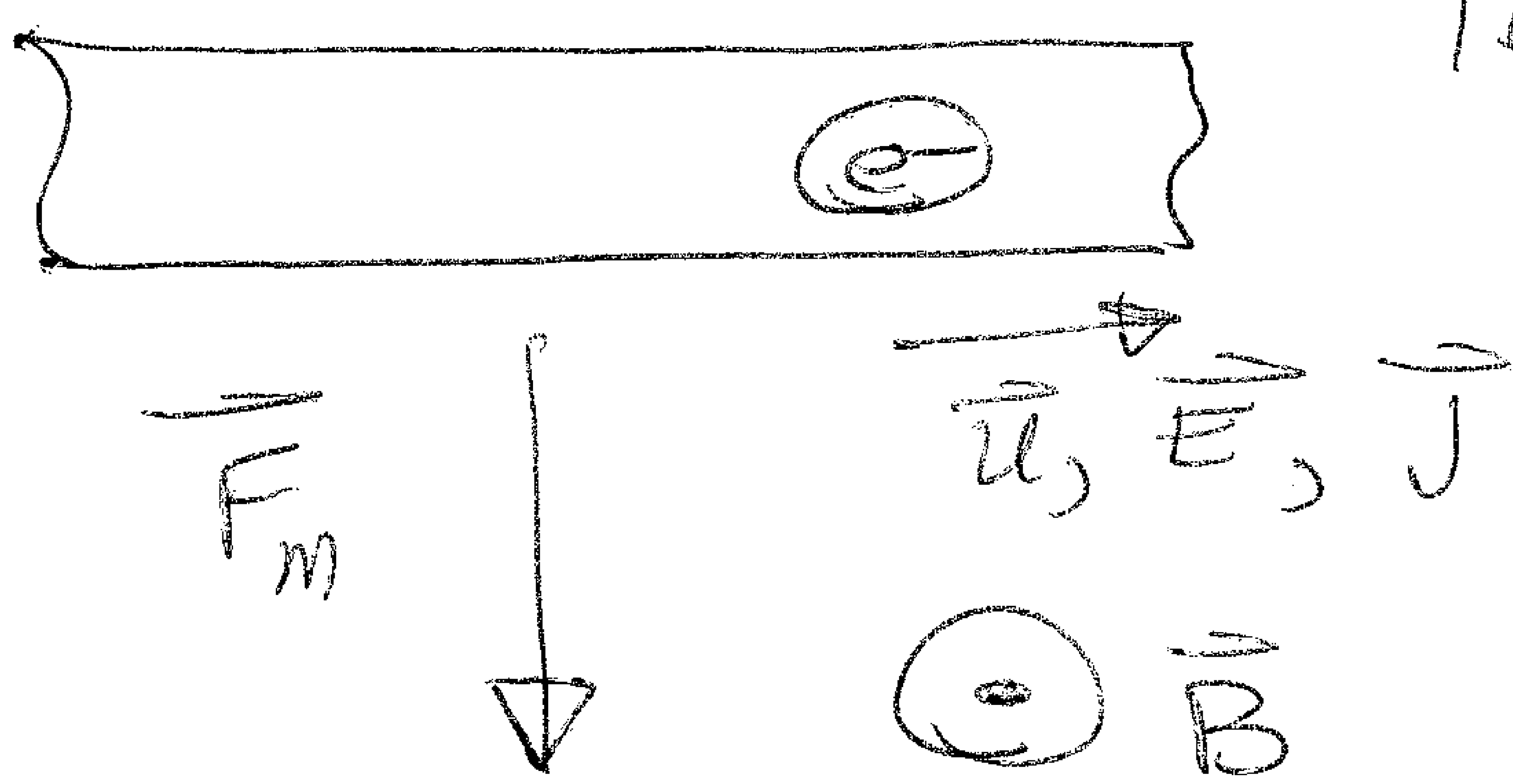
The mag force F_{mag} is normal to loop



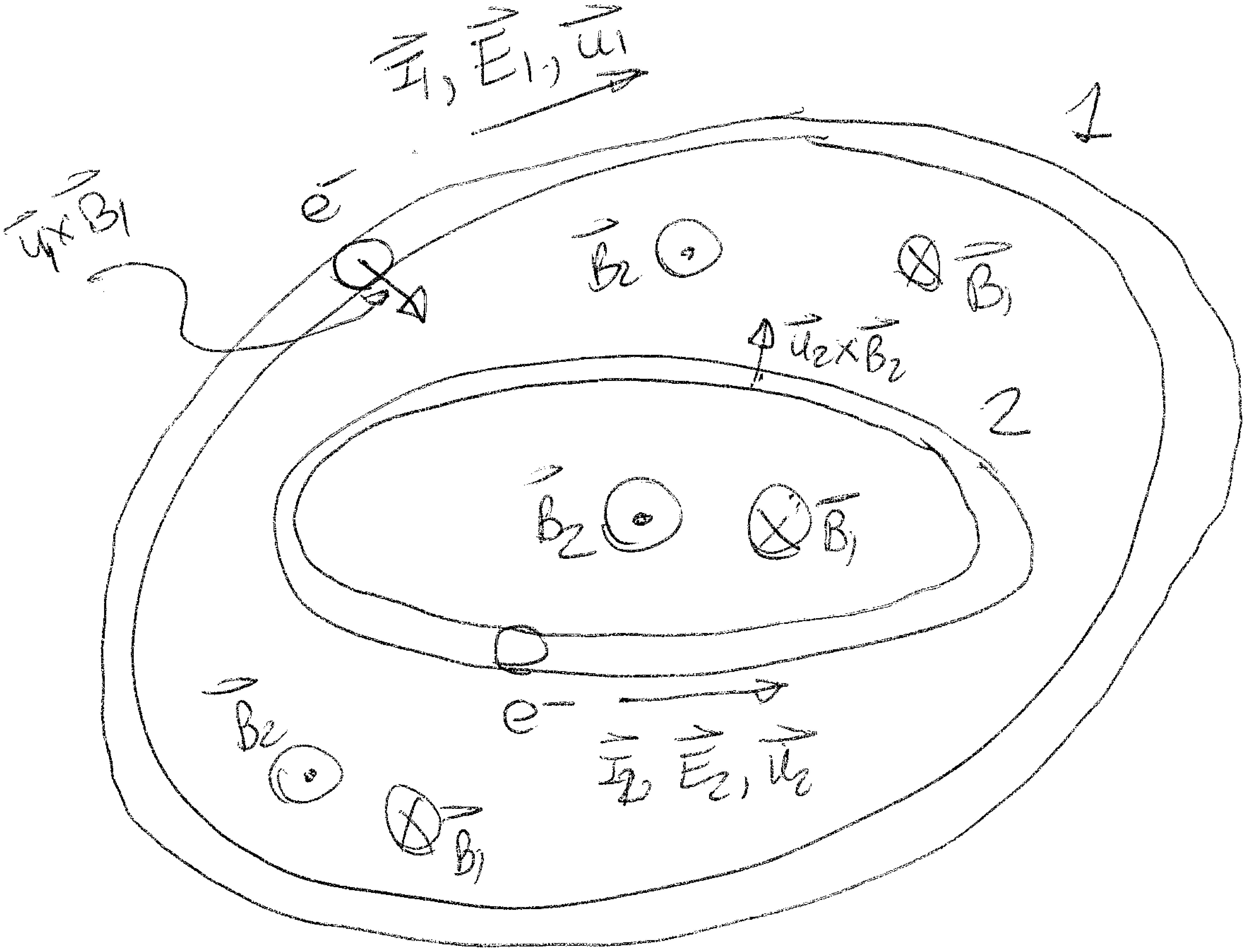
$\vec{F}_m$ tends to spin the loop



looking down onto top of loop



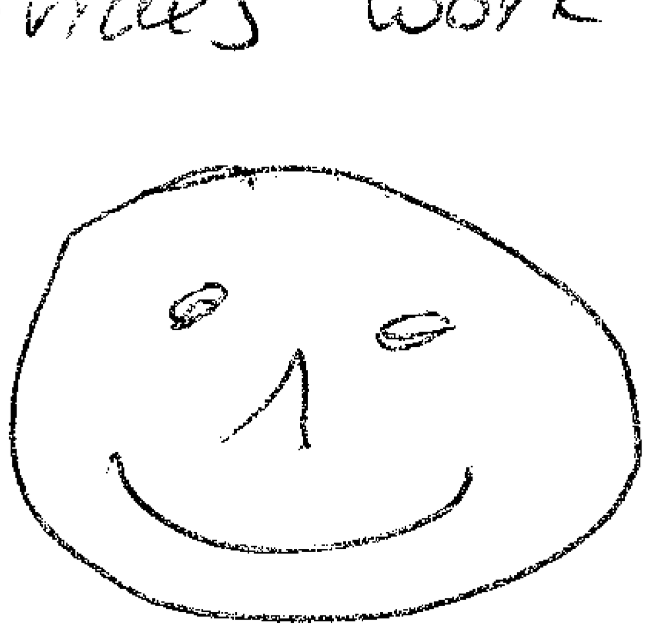
The $\vec{F}_m$ is the torque producing force.



A rough sketch of the vectors & their orientation.

All torque producing forces are the result of $\vec{u} \times \vec{B}$.

$\vec{E}$ provides work to create/maintain loop currents.



Best regards
Claude Abraham