In this part of the question we use the notation

$$\|a\|_p = \left(\sum_{k=1}^{\infty} |a_k|^p \right)^{1/p} \quad \text{and} \quad \|a\|_\infty = \sup\{|a_k| : k \in \mathbb{N}\},$$

for sequences $a = (a_k) \in l_p$ and $a = (a_k) \in l_\infty$, respectively.

(i) Show that if $a = (a_n) \in l_r, (r > 0)$, then $\|a\|_\infty = |a_J|$ for some index J.

(ii) Show that if $p < q$, then $l_p \subset l_q \subset l_\infty$.

(iii) Deduce that if $D = \bigcup_{p \geq 1} l_p$, then

$$\|a\|_\infty = \lim_{p \to \infty} \|a\|_p \text{ for all sequences } a = (a_n) \in D.$$

(iv) Show that D, as defined in Part (iii), is not a dense subset of the metric space l_∞.