

MA2325: ASSIGNMENT 4

Date due: Monday 28th November

1. Sketch the C^1 paths $\gamma : [0, 1] \rightarrow \mathbb{C}$, $t \mapsto t + it^2$ and $[0; 1 + i]$. Then compute the following integrals.

(a) $\int_{\gamma} \operatorname{Re}(z) dz$

(b) $\int_{[0; 1+i]} \operatorname{Re}(z) dz$

(c) $\int_{\gamma} z dz$

(d) $\int_{[0; 1+i]} z dz$

2. Let $\gamma : [0, 2] \rightarrow \mathbb{C}$ be the piecewise C^1 path

$$t \mapsto \begin{cases} 2e^{\pi it} & \text{for } 0 \leq t < 1 \\ 4t - 6 & \text{for } 1 \leq t \leq 2 \end{cases}$$

- (a) Sketch the path γ .

- (b) Compute the length of γ .

- (c) What are the winding numbers for γ about the points i and $-i$?

3. Let $z_0 \in \mathbb{C}$. Prove that for $k \in \mathbb{Z}$ and any $r > 0$

$$\int_{|z-z_0|=r} (z - z_0)^k dz = \begin{cases} 2\pi i & \text{if } k = -1 \\ 0 & \text{if } k \neq -1 \end{cases}$$

4. Let T be the triangular path determined by the points 0 , $1 + i$ and i . Compute the following integrals.

(a) $\int_T (z^7 + 3z^4 - z + 2) dz$

(b) $\int_T \bar{z} dz$