§(5): *System Internal Synchronization*

In order to arrive at a test theory of special relativity we shall abandon the assumption that the values of a, b are given by their special relativistic values (3.2).

We start from the transformation

$$\begin{aligned} t &= a(v)\,T + \epsilon(v)\,x \\ x &= b(v)\,(X - vT) \end{aligned} \tag{5.1}$$

where $a(v)$ and $b(v)$ remain unspecified at first.

Our first task will be to determine the values $\epsilon(v)$ corresponding to various conventions about clock synchronization. Since no relativity principle will be assumed to be valid in this section, there will be in general only one preferred ether frame Σ, in which synchronization by slow clock transport and by the Einstein procedure will agree.

Consider a clock in the system S, [S is the (t, x) system] moving with a small velocity u along the x axis. This clock will be used (in the limit of infinitesimal u) to synchronize all other clocks in S. Using $x = ut$ and (5.1) we obtain

$$X = T\left[\frac{au}{b(1 - \epsilon u)} + v\right] \simeq T\left(\frac{au}{b} + v\right) = :wT \tag{5.2}$$

if we neglect terms of order u^2 and higher. In (5.2) w is the velocity of the clock relative to Σ. If we consider this clock to be at rest at the origin of the system $S' : (t', x')$ moving with velocity w relative to Σ we obtain for the time t' which the moving clock shows

$$t'\,|_{x'=0} = a(w)\,T + \epsilon\,(w)\,x'\,|_{x'=0} = a(w)\,T \tag{5.3}$$

Synchronizing clocks by slow clock transport means that we require $t'(x) = t(x)$. Therefore we have

$$a(v)\,T + \epsilon(v)\,x = a(w)\,T \tag{5.4}$$

where

$$x = b(v)\,(X - vT) = b(v)\,(w - v)\,T = ua(v)\,T \tag{5.5}$$

Inserting this into (5.4) we obtain

$$\begin{aligned} a(v)\,u\epsilon_T &= a(w) - a(v) \simeq \frac{a(v)\,u}{b(v)}\frac{da}{dv} \\ \epsilon_T &= \frac{1}{b(v)}\frac{da(v)}{dv} \end{aligned} \tag{5.6}$$

The index T indicates that the value of ϵ given by (5.6) refers to transport synchronization of clocks. Assuming the special relativistic values for a, b we obtain the well-known relativistic result $\epsilon = -v$.

Next we shall consider the convention for clock synchronization proposed by Einstein and we shall calculate the factor ϵ_E for this case. Consider two clocks A and B at rest in the system S as shown in Figure 3. At $t = 0$ we send a light signal from A to B, where it arrives at time t_1 and is sent back to A, where it is received at time t_2. According to the Einstein procedure we shall define $t_2 = 2t_1$.

Since the clock A is at rest at the origin of S we have according to (5.1)

$$t_2 = a(v)\,T_2 + \epsilon_E x\,\big|_{x=0} = a(v)\,T_2 \tag{5.7}$$

and furthermore

$$t_1 = a(v)\,T_1 + \epsilon_E x_1 = \tfrac{1}{2}t_2 = \tfrac{1}{2}a(v)\,T_2 \tag{5.8}$$

where $x_1 = b(v)\,(X_1 - vT_1)$. The isotropy of the propagation of light in Σ implies $X_1 = T_1$ and therefore we obtain from (5.7) and (5.8)

$$a(v)\,T_1 + \epsilon_E b(v)\,(1 - v)\,T_1 = \tfrac{1}{2}a(v)\,T_2 \tag{5.9}$$

Now consider clock A, for which $X_2 = vT_2$. Inserting this and $X_1 - X_2 = T_2 - T_1$ (propagation of light) into (5.9) we obtain

$$\epsilon_E = -\frac{va(v)}{(1 - v^2)b(v)} \tag{5.10}$$

We thus arrive at the important result that the Einstein procedure in general differs from the synchronization by clock transport. The equality of both procedures is neither trivial nor logically cogent.

If we require the equality of ϵ_T and ϵ_E we have

$$a(v) = (1 - v^2)^{1/2} \tag{5.11}$$

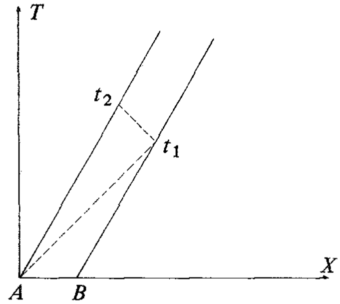

Fig. 3. Einstein synchronization of clocks.

Thus clock synchronization by clock transport and by the Einstein procedure agree if and only if the time dilatation factor is given exactly by the special relativistic value $a(v) = (1 - v^2)^{1/2}$.

§(6): *Synchronization in Three Dimensions*

We now turn to the full three-dimensional case. Here the most general linear transformation between the ether system Σ and a moving system S involves 16 coefficients, which have to be determined by kinematics, convention (synchronization), and physics. It is instructive to present the stages of simplification of the transformation in some detail, since a number of assumptions become transparent that are usually tacitly made. Our starting point is the general linear transformation

$$
\begin{aligned}
t &= aT + \epsilon x + \epsilon_2 y + \epsilon_3 z \\
x &= b_1 T + bX + b_2 Y + b_3 Z \\
y &= d_1 T + d_2 X + dY + d_3 Z \\
z &= e_1 T + e_2 X + e_3 Y + eZ
\end{aligned}
\tag{6.1}
$$

The first kinematical restriction is that the x and X axes slide along one another, i.e.,

(Kin 1) $$\forall T, X:\ y = z = 0 \longrightarrow Y = Z = 0 \tag{6.2}$$

From this we obtain

$$d_1 = d_2 = e_1 = e_2 = 0 \tag{6.3}$$

Secondly, we postulate that the (x, z) and the (X, Z) planes coincide at all times, i.e., the systems Σ and S slide along these planes:

(Kin 2) $$\forall T, X, Z:\ y = 0 \longrightarrow Y = 0 \tag{6.4}$$

whereupon we obtain

$$d_3 = 0.$$

The third requirement is that the origin of S moves with velocity v with respect to Σ:

(Kin 3) $$X = vT, \qquad Y = Z = 0 \longrightarrow x = y = z = 0 \tag{6.6}$$

This leaves us with the transformation

$$
\begin{aligned}
t &= aT + \epsilon x + \epsilon_2 y + \epsilon_3 z \\
x &= b(X - vT) + b_2 Y + b_3 Z \\
y &= dY \\
z &= eZ + e_3 Y
\end{aligned}
\tag{6.7}
$$