MATH 19A - CALCULUS I: WORKSHEET #3

To enhance your learning experience of the material in this class, weekly worksheets will be given (either on Tuesdays or Thursdays). The goal is to provide you with additional practice in addition to trying to understand things at a deeper level. A calculator may be necessary at times, especially if you're doing problems involving approximations. The number of questions on the worksheets will vary from week to week; For instance, I don't want to have you guys working on a lot of worksheet problems if you have a huge assignment due that week.

Problem 1 (Limits at Infinity)**.** Evaluate the following limits.

(a) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$

(b) $\lim_{c\to\infty} c\sqrt{\frac{c^2}{g^2}+\frac{1}{4}}-\frac{c^2}{g}$ (*Note*: g is a constant.)

(c) $\lim_{y\to\infty} \frac{\sqrt{y^2-1}-\sqrt{y^2+y-1}}{y}$

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Problem 2 (Applications of IVT)**.** Justify the following using the Intermediate Value Theorem.

(a) $f(x) = x^2 - 9$ and $g(x) = \sqrt{x+1}$ intersect on the interval $[2, 4]$.

(b) $\sqrt{2}$ exists.

Problem 3 (Bisection Method)**.** Apply the bisection method three times to Problem 2(a) to find a more accurate interval that contains the x value of the intersection point.

Problem 4 (The Definition of the Derivative)**.** Let $f(x)$ be a continuous function. We define the derivative of $f(x)$ at $x = a$, denoted $f'(a)$, to be the following:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

An equivalent definition is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

In each part, compute the derivative of $f(x)$ at $x = a$ using either definition.

(a) $f(x) = x^2 + 2x$, $x = -2$

(b) $f(x) = \sqrt{x+2}$, $x = 2$

(c) $f(x) = \dfrac{1}{\sqrt{x-1}}$, $x = 3$

(d) $f(x) = \dfrac{1}{x^2+1}$, $x = 1$