

The angular velocity of the flywheel as a function of time (and thus a function of  $\underline{n}$  (the ivt ratio from zero to  $\underline{n}_e$ ) is equal to the initial momentum of the flywheel ( $I_{fwa} \cdot \omega_{fwai}$ ) divided by the total inertia as seen by the flywheel ( $I_{fwa} + (n^2 \cdot I_{fwb})$ ):

The angular velocity of flywheel A & B per eq. 24:

The angular momentum as a function of time:

The torque is the effective change in momentum divided by the time taken to make the change:

Show that at any time ( $\underline{x}$ ) during the period  $t_i$  to  $t_e$  momentum is conserved is conserved ( $L_{fwai} = L_{fwa}(t) + L_{fwb}(t) + L_{sa}(t)$  where  $L_{sa}$  is the momentum imparted to the gear housing:

Torque applied to the IVT housing and thus the body of the satellite:

The momentum of flywheels a & B at time x:

The momentum of the satellite at time x:

Proof that that the total ending momentum is equal to the initial momentum of flywheel A:

$$(24) \quad \omega_{fwa}(t) := \frac{I_{fwa} \cdot \omega_{fwai}}{I_{fwa} + n(t)^2 \cdot I_{fwb}} \quad \omega_{fwb}(t) := n(t) \cdot \omega_{fwa}(t)$$

$$\omega_{fwa}(t_e) = 5.04 \times 10^3 \cdot \text{rpm} \quad \omega_{fwb}(t_e) = 377.974 \cdot \text{rpm}$$

$$L_{fwa}(t) := I_{fwa} \cdot \omega_{fwa}(t) \quad L_{fwb}(t) := I_{fwb} \cdot \omega_{fwb}(t)$$

$$\tau_{fwa}(t) := I_{fwa} \cdot \left( \frac{\omega_{fwa}(t) - \omega_{fwai}}{t} \right) \quad \tau_{fwb}(t) := I_{fwb} \cdot \left( \frac{\omega_{fwb}(t) - \omega_{fwb_i}}{t} \right)$$

$$x := 10 \cdot s$$

$$n(x) = 0.054$$

$$\tau_{fwa}(x) = -819.7 \cdot \text{N} \cdot \text{m}$$

$$\tau_{fwb}(x) = 1.53 \times 10^4 \cdot \text{N} \cdot \text{m}$$

$$\tau_{sa}(t) := -\tau_{fwa}(t) - \tau_{fwb}(t)$$

$$\tau_{sa}(x) = -1.448 \times 10^4 \cdot \text{N} \cdot \text{m}$$

$$L_{fwa}(x) = 1.632 \times 10^4 \frac{\text{m}^2 \cdot \text{kg}}{\text{s}}$$

$$L_{fwb}(x) = 1.53 \times 10^5 \frac{\text{m}^2 \cdot \text{kg}}{\text{s}}$$

$$L_{sa}(t) := \tau_{sa}(t) \cdot t$$

$$L_{sa}(x) = -1.448 \times 10^5 \frac{\text{m}^2 \cdot \text{kg}}{\text{s}}$$

$$L_{tot}(t) := L_{fwa}(t) + L_{fwb}(t) + L_{sa}(t)$$

$$L_{tot}(x) = 2.452 \times 10^4 \frac{\text{m}^2 \cdot \text{kg}}{\text{s}}$$

$$L_{fwai} := I_{fwa} \cdot \omega_{fwai}$$

$$L_{fwai} = 2.452 \times 10^4 \frac{\text{m}^2 \cdot \text{kg}}{\text{s}}$$

Choose any value for  $I_{sa}$  - this will result in a specific angular velocity of  $\omega_{sa}$  and will change  $EK_{sa}$  by its square. Thus, kinetic energy varies and is not conserved

$$I_{sa} := 10^{11} \cdot m^2 \cdot kg$$

$$\omega_{sa}(t) := \frac{L_{sa}(t)}{I_{sa}}$$

$$\omega_{sa}(x) = -1.383 \times 10^{-5} \cdot rpm$$

$$x := 14 \cdot s$$

$$Ek_{fwa}(t) := .5 \cdot I_{fwa} \cdot \omega_{fwa}(t)^2$$

$$Ek_{fwa}(x) = 3.261 \times 10^6 \cdot J$$

$$Ek_{fwb}(t) := .5 \cdot I_{fwb} \cdot \omega_{fwb}(t)^2$$

$$Ek_{fwb}(x) = 3.209 \times 10^6 \cdot J$$

$$Ek_{sa}(t) := .5 \cdot I_{sa} \cdot \omega_{sa}(t)^2$$

$$Ek_{sa}(x) = 0.113 \cdot J$$

$$Ek_{tot}(t) := Ek_{fwa}(t) + Ek_{fwb}(t) + Ek_{sa}(t)$$

$$Ek_{tot}(x) = 6.47 \times 10^6 \cdot J$$

$$Ek_i := Ek_{fwa}(t_i)$$

$$Ek_i = 1.284 \times 10^7 \cdot J$$

The graph shows exactly what should be expected: As the ratio of the IVT changes from 0 to .075 (which is more conventionally 1/ratio which would make it infinity to 13.33), the angular velocity of flywheel A decreases from 10,000 rpm to 5040 rpm and its energy decrease by the square of velocity while flywheel B increases from 0 to 375 rpm and its energy increases likewise. The total energy will always be less than the initial energy. The energy of the satellite spin is negligible

