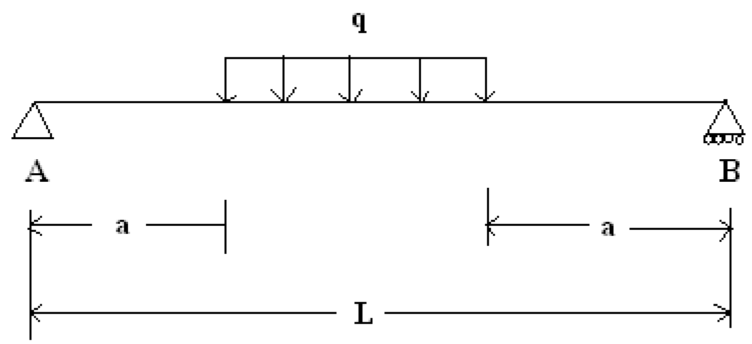
Beam Bending- Simple beam supporting a uniform load over a middle region of its span

Tried to folow Example 9.3, pg. 607 in Mechanics of Materials Sixth Edition, by James Gere



Reactions

$$R_A := \frac{q\cdot(L-2a)}{2}$$

$$R_B := \frac{q\cdot(L-2a)}{2}$$

The beam loading condition is symmetric in this case

Maximum bending moment

$$M_{max} = R_A\cdot a + \frac{1}{2}\cdot R_A\cdot\left(\frac{L-2\cdot a}{2}\right) \text{ simplify } \rightarrow M_{max} = \frac{q\cdot\left(L^2-4\cdot a^2\right)}{8}$$

Maximum shear force

$$V_{max} = \frac{q\cdot(L-2a)}{2}$$

Shear Equations

0 < x < a

$$E\cdot I\cdot\nu''' = \frac{q\cdot(L-2a)}{2}$$

a < x < L-a

$$E\cdot I\cdot\nu''' = \frac{q\cdot(L-2a)}{2} - q\cdot(x-a)$$

L-a < x < L

$$E\cdot I\cdot\nu''' = -\frac{q\cdot(L-2a)}{2}$$

Observed boundary conditions

$\nu(0) = 0$

$\nu(L) = 0$ *(Displacement at each end of the beam is zero)*

$\nu'\left(\frac{L}{2}\right) = 0$ *(Slope at the center of the beam is zero)*

$V\left(\frac{L}{2}\right) = 0$ *(Shear force is zero at the center of the beam)*

$M\left(\frac{L}{2}\right) = \frac{q\cdot\left(L^2 - 4\cdot a^2\right)}{8}$ *(This is the maximum bending moment in the beam)*

$V(0) = \frac{q\cdot(L - 2a)}{2}$

$V(L) = -\frac{q\cdot(L - 2a)}{2}$ *(Shear force at each end of the beam is defined as the maximum)*

Slopes at x=a are equal for sections 1 & 2

Deflections at x=a are equal for sections 1 & 2

Shear force and bending moments at x=a are equal for sections 1 & 2

Slopes at x=L-a are equal for sections 2 & 3

Deflections at x=L-a are equal for sections 2 & 3

Shear force and bending moments at x=L-a are equal for sections 2 & 3

Solving for the first (left-hand) section of the beam (0 < x < a)

First integration (bending moment)

$$E\cdot I\cdot\nu'' = \int \frac{q\cdot(L-2a)}{2}\,dx \rightarrow E\cdot I\cdot\nu'' = \frac{q\cdot x\cdot(L-2\cdot a)}{2}$$

$$E\cdot I\cdot\nu'' = \frac{q\cdot x\cdot(L-2\cdot a)}{2} + C_1 \;\Bigg|\begin{matrix}\text{solve},\nu''\\ \text{simplify}\end{matrix} \rightarrow \frac{C_1 + \frac{L\cdot q\cdot x}{2} - a\cdot q\cdot x}{E\cdot I}$$

Second integration (beam slope)

$$E\cdot I\cdot\nu' = \int \frac{q\cdot x\cdot(L-2\cdot a)}{2} + C_1\,dx \text{ simplify} \rightarrow E\cdot I\cdot\nu' = \frac{\left(2\cdot C_1 + L\cdot q\cdot x - 2\cdot a\cdot q\cdot x\right)^2}{4\cdot q\cdot(L-2\cdot a)}$$

$$E\cdot I\cdot\nu' = \frac{\left(2\cdot C_1 + L\cdot q\cdot x - 2\cdot a\cdot q\cdot x\right)^2}{4\cdot q\cdot(L-2\cdot a)} + C_2 \;\Bigg|\begin{matrix}\text{solve},\nu'\\ \text{simplify}\end{matrix} \rightarrow \frac{C_1^2 + C_1\cdot L\cdot q\cdot x - 2\cdot C_1\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 + C_2\cdot L\cdot q + a^2\cdot q^2\cdot x^2 - 2\cdot C_2\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)}$$

Third integration (beam deflection)

$$E\cdot I\cdot\nu = \int \frac{\left(2\cdot C_1 + L\cdot q\cdot x - 2\cdot a\cdot q\cdot x\right)^2}{4\cdot q\cdot(L-2\cdot a)} + C_2\,dx \text{ simplify} \rightarrow E\cdot I\cdot\nu = \frac{x\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot x - 12\cdot C_1\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot q\cdot(L-2\cdot a)}$$

$$E\cdot I\cdot\nu = \frac{x\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot x - 12\cdot C_1\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot q\cdot(L-2\cdot a)} \;\Bigg|\begin{matrix}\text{solve},\nu\\ \text{simplify}\end{matrix} \rightarrow \frac{x\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot x - 12\cdot C_1\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)}$$

Solving for the third (right-hand) section of the beam (L-a < x < L)

First integration (bending moment)

$$E\cdot I\cdot \nu'' = \int -\frac{q\cdot(L-2a)}{2}\,dx \rightarrow E\cdot I\cdot \nu'' = -\frac{q\cdot x\cdot(L-2\cdot a)}{2}$$

$$E\cdot I\cdot \nu'' = -\frac{q\cdot x\cdot(L-2\cdot a)}{2} + C_4 \;\Big|\begin{smallmatrix}\text{solve}, \nu''\\ \text{simplify}\end{smallmatrix} \rightarrow \frac{C_4 - \frac{L\cdot q\cdot x}{2} + a\cdot q\cdot x}{E\cdot I}$$

Second integration (beam slope)

$$E\cdot I\cdot \nu' = \int -\frac{q\cdot x\cdot(L-2\cdot a)}{2} + C_4\,dx \text{ simplify } \rightarrow E\cdot I\cdot \nu' = -\frac{(2\cdot C_4 - L\cdot q\cdot x + 2\cdot a\cdot q\cdot x)^2}{4\cdot q\cdot(L-2\cdot a)}$$

$$E\cdot I\cdot \nu' = -\frac{(2\cdot C_4 - L\cdot q\cdot x + 2\cdot a\cdot q\cdot x)^2}{4\cdot q\cdot(L-2\cdot a)} + C_5 \;\Big|\begin{smallmatrix}\text{solve}, \nu'\\ \text{simplify}\end{smallmatrix} \rightarrow -\frac{C_4^2 - C_4\cdot L\cdot q\cdot x + 2\cdot C_4\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 - C_5\cdot L\cdot q + a^2\cdot q^2\cdot x^2 + 2\cdot C_5\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)}$$

Third integration (beam deflection)

$$E\cdot I\cdot \nu = \int -\frac{(2\cdot C_4 - L\cdot q\cdot x + 2\cdot a\cdot q\cdot x)^2}{4\cdot q\cdot(L-2\cdot a)} + C_5\,dx \text{ simplify } \rightarrow E\cdot I\cdot \nu = -\frac{x\cdot\left(12\cdot C_4^2 - 6\cdot C_4\cdot L\cdot q\cdot x + 12\cdot C_4\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 - 12\cdot C_5\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 + 24\cdot C_5\cdot a\cdot q\right)}{12\cdot q\cdot(L-2\cdot a)}$$

$$E\cdot I\cdot \nu = -\frac{x\cdot\left(12\cdot C_4^2 - 6\cdot C_4\cdot L\cdot q\cdot x + 12\cdot C_4\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 - 12\cdot C_5\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 + 24\cdot C_5\cdot a\cdot q\right)}{12\cdot q\cdot(L-2\cdot a)} + C_6 \;\Big|\begin{smallmatrix}\text{solve}, \nu\\ \text{simplify}\end{smallmatrix} \rightarrow -\frac{12\cdot C_4^2\cdot x - 6\cdot C_4\cdot L\cdot q\cdot x^2 + 12\cdot C_4\cdot a\cdot q\cdot x^2 + L^2\cdot q^2\cdot x^3 - 4\cdot L\cdot a\cdot q^2\cdot x^3 - 12\cdot C_5\cdot L\cdot q\cdot x - 12\cdot C_6\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^3 + 24\cdot C_5\cdot a\cdot q\cdot x + 24\cdot C_6\cdot a\cdot q}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)}$$

Solving for middle section of beam (a < x < L-a)

First integration (bending moment)

$$E\cdot I\cdot \nu'' = \int \frac{q\cdot(L-2a)}{2} - q\cdot(x-a)\,dx \text{ simplify } \rightarrow E\cdot I\cdot \nu'' = -\frac{q\cdot(L-2\cdot x)^2}{8}$$

$$E\cdot I\cdot \nu'' = -\frac{q\cdot(L-2\cdot x)^2}{8} + C_7 \;\Bigg|\begin{matrix}\text{solve}, \nu'' \\ \text{simplify}\end{matrix} \rightarrow \frac{\frac{q\cdot L\cdot x}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot x^2}{2} + C_7}{E\cdot I}$$

Second integration (beam slope)

$$E\cdot I\cdot \nu' = \int -\frac{q\cdot(L-2\cdot x)^2}{8} + C_7\,dx \text{ simplify } \rightarrow E\cdot I\cdot \nu' = \frac{x\cdot\left(6\cdot q\cdot L\cdot x - 3\cdot q\cdot L^2 - 4\cdot q\cdot x^2 + 24\cdot C_7\right)}{24}$$

$$E\cdot I\cdot \nu' = \frac{x\cdot\left(6\cdot q\cdot L\cdot x - 3\cdot q\cdot L^2 - 4\cdot q\cdot x^2 + 24\cdot C_7\right)}{24} + C_8 \;\Bigg|\begin{matrix}\text{solve}, \nu' \\ \text{simplify}\end{matrix} \rightarrow \frac{\frac{q\cdot L\cdot x^2}{4} - \frac{q\cdot L^2\cdot x}{8} - \frac{q\cdot x^3}{6} + C_7\cdot x + C_8}{E\cdot I}$$

Third integration (deflection)

$$E\cdot I\cdot \nu = \int \frac{x\cdot\left(6\cdot q\cdot L\cdot x - 3\cdot q\cdot L^2 - 4\cdot q\cdot x^2 + 24\cdot C_7\right)}{24} + C_8\,dx \text{ simplify } \rightarrow E\cdot I\cdot \nu = \frac{q\cdot L\cdot x^3}{12} - \frac{q\cdot L^2\cdot x^2}{16} - \frac{q\cdot x^4}{24} + \frac{C_7\cdot x^2}{2} + C_8\cdot x$$

$$E\cdot I\cdot \nu = \frac{q\cdot L\cdot x^3}{12} - \frac{q\cdot L^2\cdot x^2}{16} - \frac{q\cdot x^4}{24} + \frac{C_7\cdot x^2}{2} + C_8\cdot x + C_9 \;\Bigg|\begin{matrix}\text{solve}, \nu \\ \text{simplify}\end{matrix} \rightarrow \frac{4\cdot q\cdot L\cdot x^3 - 3\cdot q\cdot L^2\cdot x^2 - 2\cdot q\cdot x^4 + 24\cdot C_7\cdot x^2 + 48\cdot C_8\cdot x + 48\cdot C_9}{48\cdot E\cdot I}$$

Solving for the unknown constants of integration C1 - C9

(0 < x < a)

$$\nu_1(x) = \frac{x\cdot\left(12\cdot C_1{}^2 + 6\cdot C_1\cdot L\cdot q\cdot x - 12\cdot C_1\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot E\cdot I\cdot q\cdot (L - 2\cdot a)}$$

$$\nu'_1(x) = \frac{C_1{}^2 + C_1\cdot L\cdot q\cdot x - 2\cdot C_1\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 + C_2\cdot L\cdot q + a^2\cdot q^2\cdot x^2 - 2\cdot C_2\cdot a\cdot q}{E\cdot I\cdot q\cdot (L - 2\cdot a)}$$

$$\nu''_1(x) = \frac{C_1 + \frac{L\cdot q\cdot x}{2} - a\cdot q\cdot x}{E\cdot I}$$

$$\nu'''_1(x) = \frac{q\cdot (L - 2a)}{2\cdot E\cdot I}$$

(a < x < L - a)

$$\nu_2(x) = \frac{4\cdot q\cdot L\cdot x^3 - 3\cdot q\cdot L^2\cdot x^2 - 2\cdot q\cdot x^4 + 24\cdot C_7\cdot x^2 + 48\cdot C_8\cdot x + 48\cdot C_9}{48\cdot E\cdot I}$$

$$\nu'_2(x) = \frac{\frac{q\cdot L\cdot x^2}{4} - \frac{q\cdot L^2\cdot x}{8} - \frac{q\cdot x^3}{6} + C_7\cdot x + C_8}{E\cdot I}$$

$$\nu''_2(x) = \frac{\frac{q\cdot L\cdot x}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot x^2}{2} + C_7}{E\cdot I}$$

$$\nu'''_2(x) = \frac{\frac{q\cdot (L - 2a)}{2} - q\cdot (x - a)}{E\cdot I}$$

(L - a < x < L)

$$\nu_3(x) = -\frac{12\cdot C_4{}^2\cdot x - 6\cdot C_4\cdot L\cdot q\cdot x^2 + 12\cdot C_4\cdot a\cdot q\cdot x^2 + L^2\cdot q^2\cdot x^3 - 4\cdot L\cdot a\cdot q^2\cdot x^3 - 12\cdot C_5\cdot L\cdot q\cdot x - 12\cdot C_6\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^3 + 24\cdot C_5\cdot a\cdot q\cdot x + 24\cdot C_6\cdot a\cdot q}{12\cdot E\cdot I\cdot q\cdot (L - 2\cdot a)}$$

$$\nu'_3(x) = -\frac{C_4{}^2 - C_4\cdot L\cdot q\cdot x + 2\cdot C_4\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 - C_5\cdot L\cdot q + a^2\cdot q^2\cdot x^2 + 2\cdot C_5\cdot a\cdot q}{E\cdot I\cdot q\cdot (L - 2\cdot a)}$$

$$\nu''_3(x) = \frac{C_4 - \frac{L\cdot q\cdot x}{2} + a\cdot q\cdot x}{E\cdot I}$$

$$\nu'''_3(x) = -\frac{q\cdot (L - 2a)}{2E\cdot I}$$

Given

The deflection at the joining points of each section are equal:

$$\frac{a\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot a - 12\cdot C_1\cdot a\cdot q\cdot a + L^2\cdot q^2\cdot a^2 - 4\cdot L\cdot a\cdot q^2\cdot a^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot a^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)} = \frac{4\cdot q\cdot L\cdot a^3 - 3\cdot q\cdot L^2\cdot a^2 - 2\cdot q\cdot a^4 + 24\cdot C_7\cdot a^2 + 48\cdot C_8\cdot a + 48\cdot C_9}{48\cdot E\cdot I}$$

$$\frac{4\cdot q\cdot L\cdot(L-a)^3 - 3\cdot q\cdot L^2\cdot(L-a)^2 - 2\cdot q\cdot(L-a)^4 + 24\cdot C_7\cdot(L-a)^2 + 48\cdot C_8\cdot(L-a) + 48\cdot C_9}{48\cdot E\cdot I} = -\frac{12\cdot C_4^2\cdot(L-a) - 6\cdot C_4\cdot L\cdot q\cdot(L-a)^2 + 12\cdot C_4\cdot a\cdot q\cdot(L-a)^2 + L^2\cdot q^2\cdot(L-a)^3 - 4\cdot L\cdot a\cdot q^2\cdot(L-a)^3 - 12\cdot C_5\cdot L\cdot q\cdot(L-a) - 12\cdot C_6\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot(L-a)^3 + 24\cdot C_5\cdot a\cdot q\cdot(L-a) + 24\cdot C_6\cdot a\cdot q}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)}$$

The slope at the joining points of each section are equal:

$$\frac{C_1^2 + C_1\cdot L\cdot q\cdot a - 2\cdot C_1\cdot a\cdot q\cdot a + \frac{L^2\cdot q^2\cdot a^2}{4} - L\cdot a\cdot q^2\cdot a^2 + C_2\cdot L\cdot q + a^2\cdot q^2\cdot a^2 - 2\cdot C_2\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)} = \frac{\frac{q\cdot L\cdot a^2}{4} - \frac{q\cdot L^2\cdot a}{8} - \frac{q\cdot a^3}{6} + C_7\cdot a + C_8}{E\cdot I}$$

$$\frac{\frac{q\cdot L\cdot(L-a)^2}{4} - \frac{q\cdot L^2\cdot(L-a)}{8} - \frac{q\cdot(L-a)^3}{6} + C_7\cdot(L-a) + C_8}{E\cdot I} = -\frac{C_4^2 - C_4\cdot L\cdot q\cdot(L-a) + 2\cdot C_4\cdot a\cdot q\cdot(L-a) + \frac{L^2\cdot q^2\cdot(L-a)^2}{4} - L\cdot a\cdot q^2\cdot(L-a)^2 - C_5\cdot L\cdot q + a^2\cdot q^2\cdot(L-a)^2 + 2\cdot C_5\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)}$$

The bending moment at the joining points of each section are equal

$$C_1 + \frac{L\cdot q\cdot a}{2} - a\cdot q\cdot a = \frac{q\cdot L\cdot a}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot a^2}{2} + C_7$$

$$\frac{q\cdot L\cdot(L-a)}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot(L-a)^2}{2} + C_7 = C_4 - \frac{L\cdot q\cdot(L-a)}{2} + a\cdot q\cdot(L-a)$$

Deflection at each end is zero

$$\frac{0\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot 0 - 12\cdot C_1\cdot a\cdot q\cdot 0 + L^2\cdot q^2\cdot 0^2 - 4\cdot L\cdot a\cdot q^2\cdot 0^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot 0^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)} = 0$$

$$-\frac{12\cdot C_4^2\cdot L - 6\cdot C_4\cdot L\cdot q\cdot L^2 + 12\cdot C_4\cdot a\cdot q\cdot L^2 + L^2\cdot q^2\cdot L^3 - 4\cdot L\cdot a\cdot q^2\cdot L^3 - 12\cdot C_5\cdot L\cdot q\cdot L - 12\cdot C_6\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot L^3 + 24\cdot C_5\cdot a\cdot q\cdot L + 24\cdot C_6\cdot a\cdot q}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)} = 0$$

Bending Moment in the middle is maximum

$$\frac{q\cdot L\cdot\frac{L}{2}}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot\left(\frac{L}{2}\right)^2}{2} + C_7 = \frac{q\cdot\left(L^2 - 4\cdot a^2\right)}{8}$$

$$\begin{pmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \\ C_9 \end{pmatrix} := \mathrm{Find}(C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9) \rightarrow \text{ simplify } \rightarrow \begin{bmatrix} 0 \\ -\frac{q\cdot\left(L^3 - 6\cdot L\cdot a^2 + 4\cdot a^3\right)}{24} \\ 0 \\ \frac{L\cdot q\cdot(L - 2\cdot a)}{2} \\ \frac{q\cdot\left(L^3 - 6\cdot L\cdot a^2 + 4\cdot a^3\right)}{24} \\ \frac{L\cdot q\cdot(L - 2\cdot a)\cdot\left(L^2 - 2\cdot L\cdot a + 2\cdot a^2\right)}{24} \\ \frac{q\cdot\left(L^2 - 4\cdot a^2\right)}{8} \\ -\frac{L\cdot q\cdot\left(L^2 - 6\cdot a^2\right)}{24} \\ -\frac{a^4\cdot q}{24} \end{bmatrix}$$

SOLUTIONS

$\nu_n(x)$ *Deflection*

$\nu'_n(x)$ *Angle*

$M_n(x)$ Moment

(0 < x < a)

$$\nu_1(x) = \frac{x\cdot\left(12\cdot C_1^2 + 6\cdot C_1\cdot L\cdot q\cdot x - 12\cdot C_1\cdot a\cdot q\cdot x + L^2\cdot q^2\cdot x^2 - 4\cdot L\cdot a\cdot q^2\cdot x^2 + 12\cdot C_2\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^2 - 24\cdot C_2\cdot a\cdot q\right)}{12\cdot E\cdot I\cdot q\cdot(L - 2\cdot a)} \text{ simplify } \rightarrow \nu_1(x) = -\frac{q\cdot x\cdot(L-2\cdot a)\cdot\left(L^2 + 2\cdot L\cdot a - 2\cdot a^2 - 2\cdot x^2\right)}{24\cdot E\cdot I}$$

$$\nu'_1(x) = \frac{C_1^2 + C_1\cdot L\cdot q\cdot x - 2\cdot C_1\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 + C_2\cdot L\cdot q + a^2\cdot q^2\cdot x^2 - 2\cdot C_2\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)} \text{ simplify } \rightarrow \nu'_1(x) = -\frac{q\cdot(L-2\cdot a)\cdot\left(L^2 + 2\cdot L\cdot a - 2\cdot a^2 - 6\cdot x^2\right)}{24\cdot E\cdot I}$$

$$M_1(x) = C_1 + \frac{L\cdot q\cdot x}{2} - a\cdot q\cdot x \text{ simplify } \rightarrow M_1(x) = \frac{q\cdot x\cdot(L-2\cdot a)}{2}$$

(a < x < L - a)

$$\nu_2(x) = \frac{4\cdot q\cdot L\cdot x^3 - 3\cdot q\cdot L^2\cdot x^2 - 2\cdot q\cdot x^4 + 24\cdot C_7\cdot x^2 + 48\cdot C_8\cdot x + 48\cdot C_9}{48\cdot E\cdot I} \text{ simplify } \rightarrow \nu_2(x) = -\frac{q\cdot L^3\cdot x - 6\cdot q\cdot L\cdot a^2\cdot x - 2\cdot q\cdot L\cdot x^3 + q\cdot a^4 + 6\cdot q\cdot a^2\cdot x^2 + q\cdot x^4}{24\cdot E\cdot I}$$

$$\nu'_2(x) = \frac{\frac{q\cdot L\cdot x^2}{4} - \frac{q\cdot L^2\cdot x}{8} - \frac{q\cdot x^3}{6} + C_7\cdot x + C_8}{E\cdot I} \text{ simplify } \rightarrow \nu'_2(x) = -\frac{q\cdot(L-2\cdot x)\cdot\left(L^2 + 2\cdot L\cdot x - 6\cdot a^2 - 2\cdot x^2\right)}{24\cdot E\cdot I}$$

$$M_2(x) = \frac{q\cdot L\cdot x}{2} - \frac{q\cdot L^2}{8} - \frac{q\cdot x^2}{2} + C_7 \text{ simplify } \rightarrow M_2(x) = -\frac{q\cdot\left(a^2 + x^2 - L\cdot x\right)}{2}$$

(L - a < x < L)

$$\nu_3(x) = -\frac{12\cdot C_4^2\cdot x - 6\cdot C_4\cdot L\cdot q\cdot x^2 + 12\cdot C_4\cdot a\cdot q\cdot x^2 + L^2\cdot q^2\cdot x^3 - 4\cdot L\cdot a\cdot q^2\cdot x^3 - 12\cdot C_5\cdot L\cdot q\cdot x - 12\cdot C_6\cdot L\cdot q + 4\cdot a^2\cdot q^2\cdot x^3 + 24\cdot C_5\cdot a\cdot q\cdot x + 24\cdot C_6\cdot a\cdot q}{12\cdot E\cdot I\cdot q\cdot(L-2\cdot a)} \text{ simplify } \rightarrow \nu_3(x) = \frac{q\cdot(L-2\cdot a)\cdot(L-x)\cdot\left(L^2 - 2\cdot L\cdot a - 4\cdot L\cdot x + 2\cdot a^2 + 2\cdot x^2\right)}{24\cdot E\cdot I}$$

$$\nu'_3(x) = -\frac{C_4^2 - C_4\cdot L\cdot q\cdot x + 2\cdot C_4\cdot a\cdot q\cdot x + \frac{L^2\cdot q^2\cdot x^2}{4} - L\cdot a\cdot q^2\cdot x^2 - C_5\cdot L\cdot q + a^2\cdot q^2\cdot x^2 + 2\cdot C_5\cdot a\cdot q}{E\cdot I\cdot q\cdot(L-2\cdot a)} \text{ simplify } \rightarrow \nu'_3(x) = -\frac{q\cdot(L-2\cdot a)\cdot\left(5\cdot L^2 - 2\cdot L\cdot a - 12\cdot L\cdot x + 2\cdot a^2 + 6\cdot x^2\right)}{24\cdot E\cdot I}$$

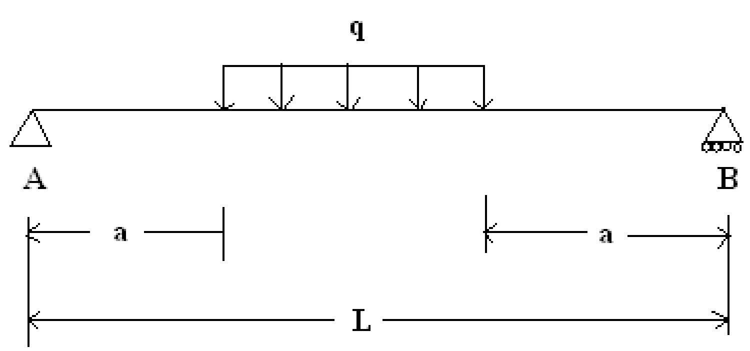
$$M_3(x) = C_4 - \frac{L\cdot q\cdot x}{2} + a\cdot q\cdot x \text{ simplify } \rightarrow M_3(x) = \frac{q\cdot(L-2\cdot a)\cdot(L-x)}{2}$$

$$\delta_{max} = -\frac{q\cdot L^3\cdot x - 6\cdot q\cdot L\cdot a^2\cdot x - 2\cdot q\cdot L\cdot x^3 + q\cdot a^4 + 6\cdot q\cdot a^2\cdot x^2 + q\cdot x^4}{24\cdot E\cdot I} \left|\begin{array}{l}\text{substitute}, x = \frac{L}{2}\\ \text{simplify}\end{array}\right. \rightarrow \delta_{max} = -\frac{5\cdot q\cdot L^4 - 24\cdot q\cdot L^2\cdot a^2 + 16\cdot q\cdot a^4}{384\cdot E\cdot I}$$

$$M_{max} = -\frac{q\cdot\left(a^2 + x^2 - L\cdot x\right)}{2} \left|\begin{array}{l}\text{substitute}, x = \frac{L}{2}\\ \text{simplify}\end{array}\right. \rightarrow M_{max} = \frac{q\cdot\left(L^2 - 4\cdot a^2\right)}{8}$$

$$\theta_A = -\frac{q\cdot(L - 2\cdot a)\cdot\left(L^2 + 2\cdot L\cdot a - 2\cdot a^2 - 6\cdot x^2\right)}{24\cdot E\cdot I} \left|\begin{array}{l}\text{substitute}, x = 0\\ \text{simplify}\end{array}\right. \rightarrow \theta_A = -\frac{q\cdot L^3 - 6\cdot q\cdot L\cdot a^2 + 4\cdot q\cdot a^3}{24\cdot E\cdot I}$$

Sample Problem



$E := 200GPa$ $\qquad q := 2000\frac{N}{m}$ $\qquad L := 5m$ $\qquad a := 1m$

$b := 5cm$ $\qquad h := 15cm$

$$I := \frac{b\cdot h^3}{12}$$

$$M_{max} := \frac{q\cdot\left(L^2 - 4\cdot a^2\right)}{8} \qquad M_{max} = 5.25\times 10^3\cdot N\cdot m$$

$$\delta_{max} := -\frac{5\cdot q\cdot L^4 - 24\cdot q\cdot L^2\cdot a^2 + 16\cdot q\cdot a^4}{384\cdot E\cdot I} \qquad \delta_{max} = -4.706\cdot mm$$

$$\theta_A := -\frac{q\cdot L^3 - 6\cdot q\cdot L\cdot a^2 + 4\cdot q\cdot a^3}{24\cdot E\cdot I} \qquad \theta_A = -2.933\times 10^{-3}$$

From Solution Appendix in Book:

$$\theta_{Ab} := \frac{q\cdot\left(L^3 - 6\cdot L\cdot a^2 + 4\cdot a^3\right)}{24\cdot E\cdot I} \qquad \theta_{Ab} = 2.933\times 10^{-3}$$

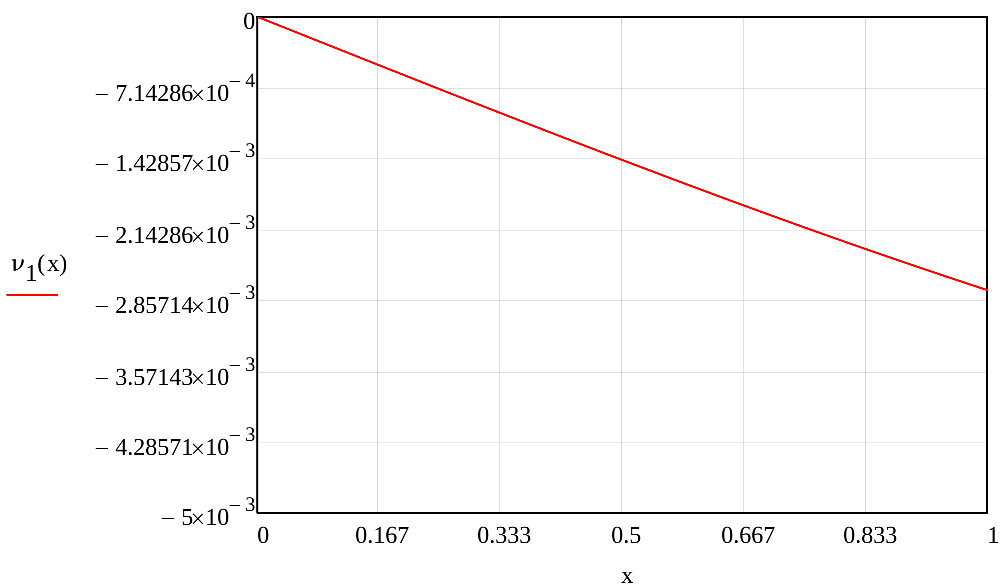
$$\delta_{maxb} := \frac{q\cdot\left(5\cdot L^4 - 24\cdot L^2\cdot a^2 + 16\cdot a^4\right)}{384\cdot E\cdot I} \qquad \delta_{maxb} = 4.706\cdot mm$$

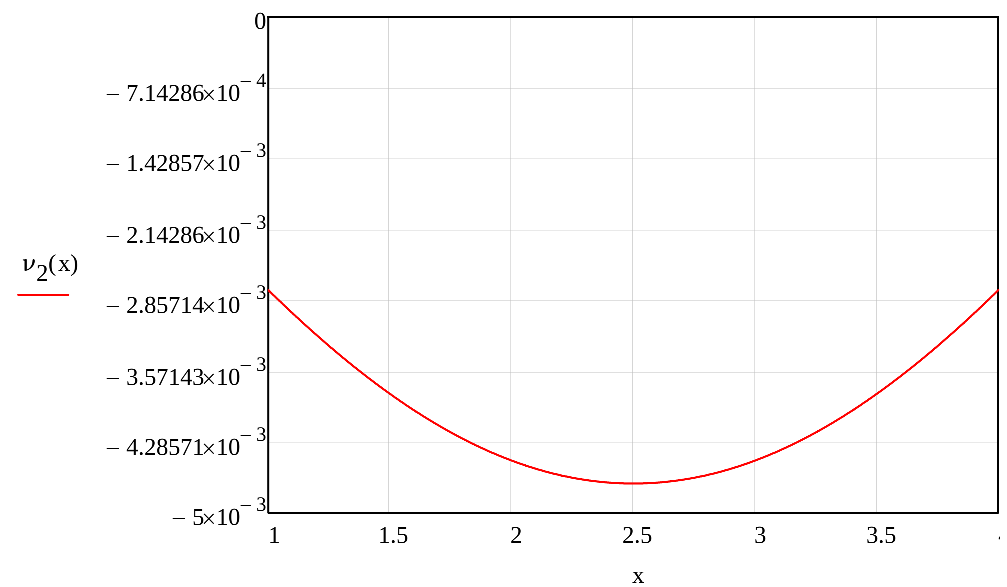
Beam Deflection Graphs

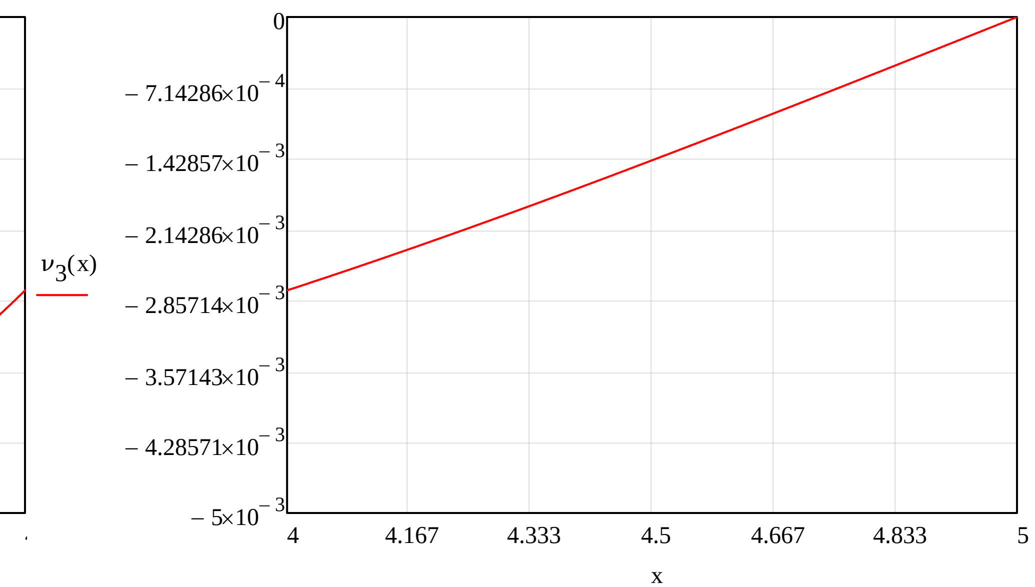
$$\nu_1(x) := -\frac{q\cdot x\cdot(L-2\cdot a)\cdot\left(L^2+2\cdot L\cdot a-2\cdot a^2-2\cdot x^2\right)}{24\cdot E\cdot I}$$

$$\nu_2(x) := -\frac{q\cdot L^3\cdot x-6\cdot q\cdot L\cdot a^2\cdot x-2\cdot q\cdot L\cdot x^3+q\cdot a^4+6\cdot q\cdot a^2\cdot x^2+q\cdot x^4}{24\cdot E\cdot I}$$

$$\nu_3(x) := \frac{q\cdot(L-2\cdot a)\cdot(L-x)\cdot\left(L^2-2\cdot L\cdot a-4\cdot L\cdot x+2\cdot a^2+2\cdot x^2\right)}{24\cdot E\cdot I}$$





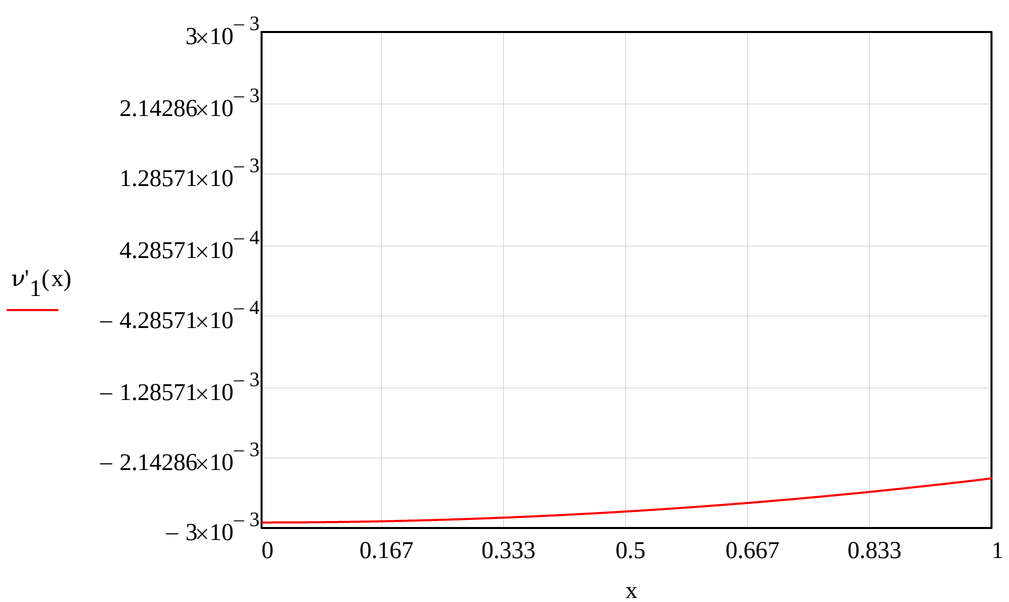


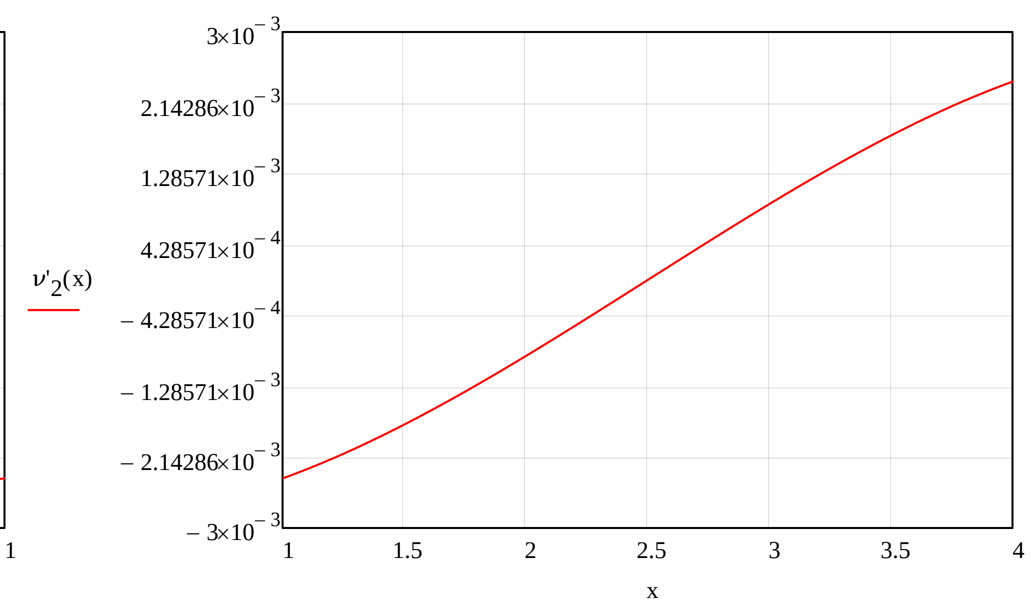
Beam Slope Graphs

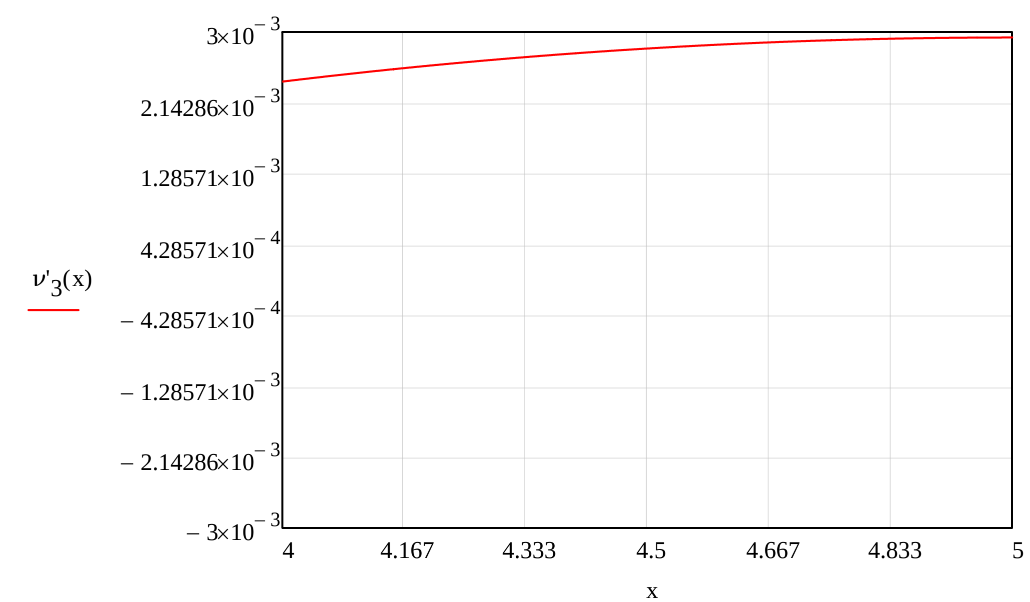
$$\nu'_1(x) := -\frac{q\cdot(L-2\cdot a)\cdot\left(L^2+2\cdot L\cdot a-2\cdot a^2-6\cdot x^2\right)}{24\cdot E\cdot I}$$

$$\nu'_2(x) := -\frac{q\cdot(L-2\cdot x)\cdot\left(L^2+2\cdot L\cdot x-6\cdot a^2-2\cdot x^2\right)}{24\cdot E\cdot I}$$

$$\nu'_3(x) := -\frac{q\cdot(L-2\cdot a)\cdot\left(5\cdot L^2-2\cdot L\cdot a-12\cdot L\cdot x+2\cdot a^2+6\cdot x^2\right)}{24\cdot E\cdot I}$$





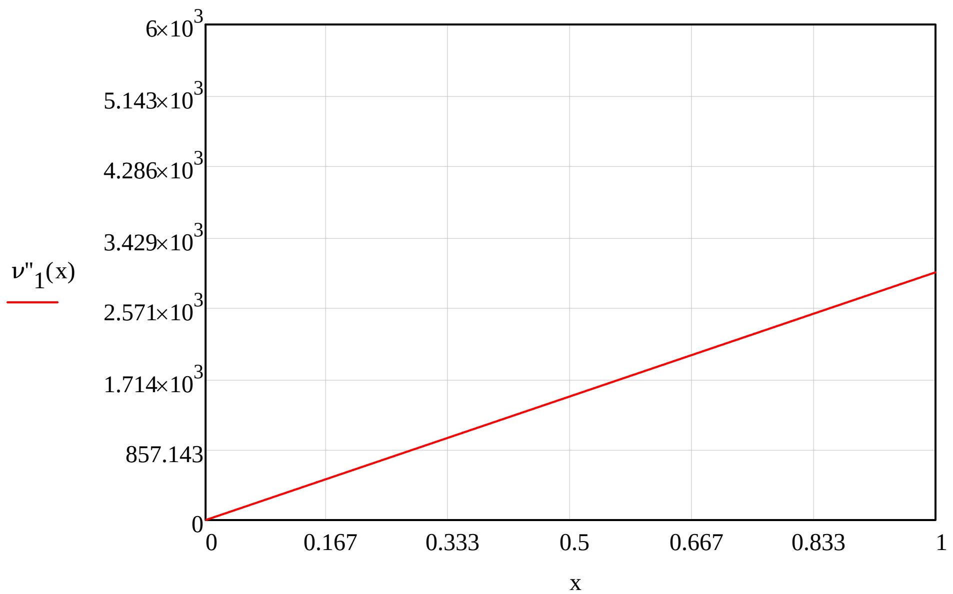


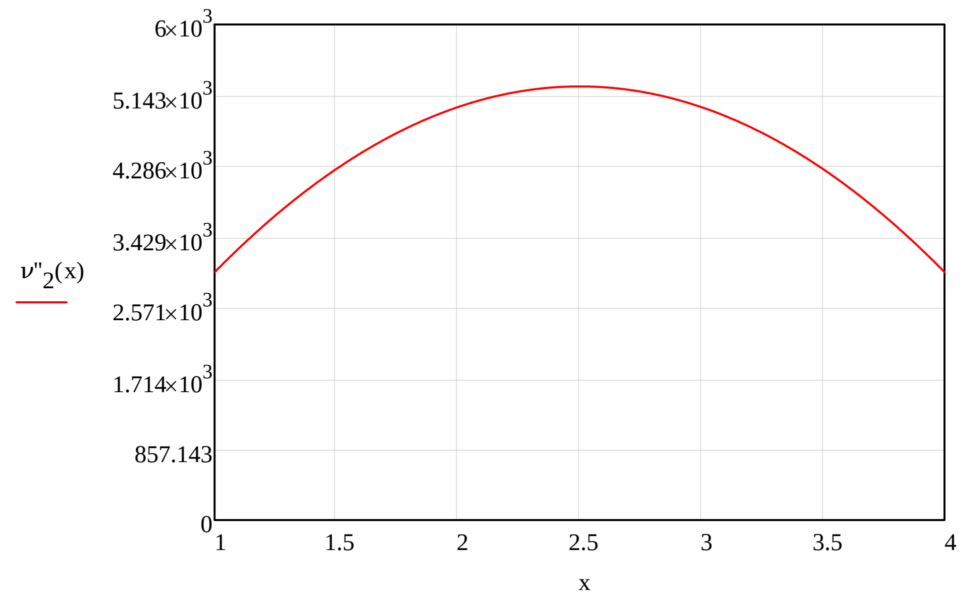
Beam Bending Moment Graphs

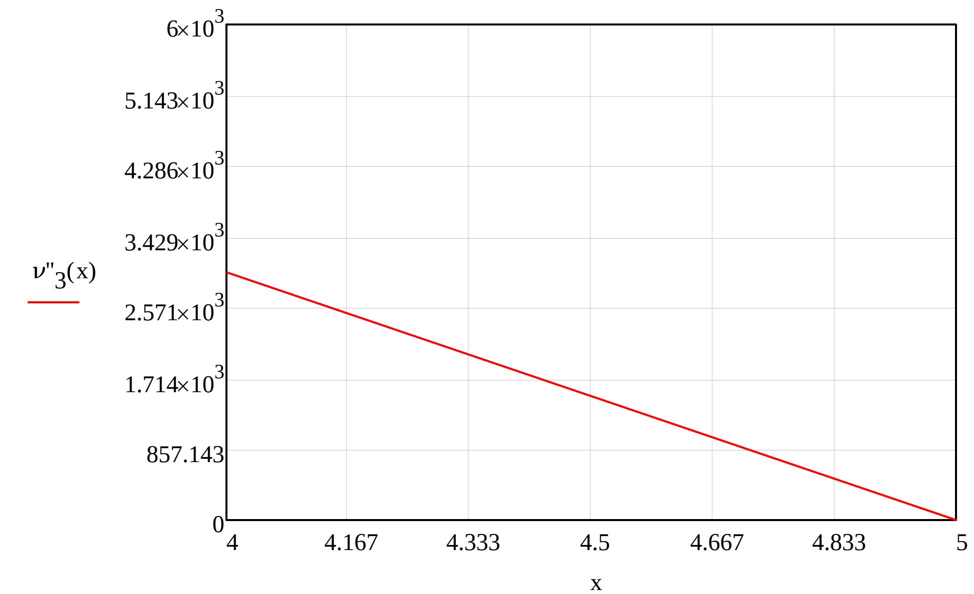
$$\nu''_1(x) := \frac{q \cdot x \cdot (L - 2 \cdot a)}{2}$$

$$\nu''_2(x) := -\frac{q \cdot \left(a^2 + x^2 - L \cdot x\right)}{2}$$

$$\nu''_3(x) := \frac{q \cdot (L - 2 \cdot a) \cdot (L - x)}{2}$$







Beam Shear Force Graphs

$$\nu'''_1(x) := \frac{q \cdot (L - 2a)}{2}$$

$$\nu'''_2(x) := \frac{q \cdot (L - 2a)}{2} - q \cdot (x - a)$$

$$\nu'''_3(x) := -\frac{q \cdot (L - 2a)}{2}$$