Mathematics Memorization Lists

1. *Mathematics* is the art of recognizing numerical patterns in God's revelation. - Michaela Farrell. In Lingua Latina: Mathēmatica est ars agnoscentis exemplāra numerōrum in revēlātiōne Deī.

2. *Arithmetic* is that branch of mathematics dealing with the study of numbers directly, particularly the properties of the traditional operations between them - addition, subtraction, multiplication, and division. - Wikipedia.

3. *Algebra*[1] is that branch of mathematics dealing with the manipulation of symbols to solve equations and inequalities.

4. *Geometry*[2] is that branch of mathematics dealing with questions of size, shape, relative position of figures, and the properties of space. - Wikipedia

5. *Analytic Geometry* is that branch of mathematics dealing with the replacement of curves with functions so that geometry becomes analysis.

6. *Trigonometry* is that branch of mathematics that studies relationships involving lengths and angles of triangles, particularly right triangles. - Wikipedia.

7. *Statistics*[3] is that branch of mathematics dealing with the collection, analysis, interpretation, presentation, and organization of data. - Wikipedia

Arithmetic

8. *Addition and Subtraction Facts* up to 20. That is, the students should be able to add any two numbers, each of which is less than 20, and the same for subtraction.

9. *Multiplication Tables* from 0 to 15. Students should be able to compute any numbers in the following table instantly, correctly, without thinking about it. Students should be able to count by x up to $x \cdot 15$. Students should be able to compute $6 \cdot 7 = 42$ from memory, as well as $7 \cdot 6 = 42$ from memory. They

[1] Algebra and Analytic Geometry Grammar items taken from *Algebra I* and *Algebra II*, by Larson et al.
[2] Geometry Grammar items taken from *Geometry*, by Larson et al.
[3] Probability and Statistics Grammar items taken from *Elementary Statistics: A Step by Step Approach, 6th Ed.*, by Bluman.

should not be using the Commutative Property to compute either of these.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	0	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	0	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	0	8	16	24	32	40	48	56	64	72	80	88	96	104	112	120
9	0	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	0	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	0	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	0	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	0	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

10. *Perfect Squares* up to 15. That is, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225.

11. A *Prime Number* is a number divisible only by two distinct numbers: 1 and itself. (Hence, 1 is not a prime number.)

12. *Prime Numbers* up to 31. That is, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31.

13. *Divisibility Rules* for 2, 3, 5, 9. Even numbers are divisible by 2, numbers are divisible by 3 if their digit sum is divisible by 3, numbers are divisible by 5 if the last digit is 0 or 5, and numbers are divisible by 9 if their digit sum is divisible by 9.

14. *Order of Operations* is PEMDAS: Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction.

15. *Trachtenberg Rule for Multiplying by* 11*:* Add the neighbor.

16. *Trachtenberg Rule for Multiplying by* 12*:* Double the number and add the neighbor.

17. *Trachtenberg Rule for Multiplying by* 6*:* Add 5 to the number if the number is odd; add nothing if it is even. Add "half" the neighbor.

18. *Trachtenberg Rule for Multiplying by* 7*:* Double the number and add "half" the neighbor; add 5 if the number is odd.

19. *Trachtenberg Rule for Multiplying by* 5*:* "Half" the neighbor, plus 5 if the number is odd.

20. *Trachtenberg Rule for Multiplying by* 9*:*

    a. Subtract the right-hand figure of the long number from ten. This gives the right-hand figure of the answer.

b. Taking each of the following figures in turn, up to the last one, subtract it from nine and add the neighbor.
c. At the last step, when you are under the zero in front of the long number, subtract one from the neighbor and use that as the left-hand figure of the answer.

21. *Trachtenberg Rule for Multiplying by* 8*:*

 a. First figure: subtract from ten and *double*.
 b. Middle figures: subtract from nine and *double* what you get, then add the neighbor.
 c. Left-hand figure: subtract *two* from the left-hand figure of the long number.

22. *Trachtenberg Rule for Multiplying by* 4*:*

 a. Subtract the right-hand digit of the given number from ten, and add five if that digit is odd.
 b. Subtract each digit of the given number in turn from nine, add five if the digit is odd, and add half the neighbor.
 c. Under the zero in front of the given number, write half the neighbor of this zero, less one.

23. *Trachtenberg Rule for Multiplying by* 3*:*

 a. First figure: subtract from ten and double. Add five if the number is odd.
 b. Middle figures: subtract the number from nine and double what you get, then add half the neighbor. Add five if the number is odd.
 c. Left-hand figure: divide the left-hand figure of the long number in *half*; then subtract two.

Algebra

24. *Closure Law of Addition:* If a and b are real numbers, then $a + b$ is a real number.

25. *Commutative Law of Addition:* $a + b = b + a$.

26. *Associative Law of Addition:* $a + (b + c) = (a + b) + c$.

27. *Additive Identity:* $a + 0 = a$.

28. *Additive Inverse:* $a + (-a) = a - a = 0$.

29. *Closure Law of Multiplication:* If a and b are real numbers, then ab is a real number.

30. *Commutative Law of Multiplication:* $ab = ba$.

31. *Associative Law of Multiplication:* $a(bc) = (ab)c$.

32. *Multiplicative Identity:* $1 \cdot a = a$.

33. *Multiplicative Inverse:* $(1/a) \cdot a = 1$, if $a \neq 0$.

34. *Distributive Law:* $a(b + c) = ab + ac$.

35. Multiplication is peculiar in distributing over addition. In particular, the *Freshman's Dream Inequality No. 1* is that $(x + y)^2 \neq x^2 + y^2$. Instead, $(x + y)^2 = x^2 + 2xy + y^2$. Exponentiation does not distribute over addition. The *Freshman's Dream Inequality No. 2* is that $\sqrt{x^2 + y^2} \neq x + y$. The expression on the left does not simplify. So, taking radicals does not distribute over addition.

36. *Addition/Subtraction Property of Equality:* If $a = b$, then $a \pm c = b \pm c$.

37. *Multiplication Property of Equality:* If $a = b$, then $ac = bc$.

38. *Division Property of Equality:* If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

39. *Substitution Property of Equality:* If $a = b$, then a can be substituted for b in any equation or expression.

40. *Reflexive Property of Equality:* For any real number a, $a = a$.

41. *Symmetric Property of Equality:* For any real numbers a and b, if $a = b$ then $b = a$.

42. *Transitive Property of Equality:* For any real numbers a, b, and c, if $a = b$ and $b = c$, then $a = c$.

43. *Solving an Equation for x* means manipulating an equation using valid laws of algebra to isolate x on one side of the equation, with no x's on the other side of the equation.

44. *The Golden Rule of Algebra:* What thou doest to one side of an equation thou must do to the other.

45. *Conservation of Symbols Law:* In any derivation, all symbols (digits, decimal points, parentheses, arithmetic signs such as $+, -, \times, =$, etc.) from one line must survive to the next line unless a specific valid algebraic property is invoked to alter them. Moreover, no new symbols may be introduced in a new line, unless a specific, valid algebraic property is invoked to do so.

46. A *term* is a collection of quantities that are multiplied together.

47. *Like terms* are terms that have the same variables raised to the same powers.

48. Given any term, the *coefficient of x in that term* is everything in that term multiplying the x.

49. The *Least Common Multiple (LCM)* of any number of terms is the smallest quantity that all the terms will divide into evenly.

50. The *Greatest Common Factor (GCF)* of any number of terms is the largest quantity that will divide evenly into all the terms.

51. The *Least Common Denominator (LCD)* of any number of fractions is the LCM of their denominators.

52. *Steps to solve a linear equation for x:* (not all steps apply to all equations):

    a. Eliminate all denominators by multiplying the entire equation (that is, every term in it) by the LCD of all fractions present.

    b. Multiply out parentheses using the Distributive Law.

    c. Bring all terms containing x to one side of the equation, and all terms not containing x to the other side of the equation.

    d. Combine any like terms.

e. Factor x out of all terms containing it, according to the Distributive Law.

f. Divide both sides of the equation by the resulting coefficient of x.

53. All *graphs* must have SOUL: **S**cale, **O**rigin, **U**nits, and **L**abels.

54. A *relation* is a mapping or pairing of input values with output values. The set of all allowed input values is the *domain*, and the set of all output values that come from the input values through the relation is the *range*. The *rule of association* is the statement explaining how the input values get mapped to the output values.

55. A *function* is a relation for which each input has exactly one output.

56. If *y varies directly as x*, then $y = ax$ for some constant a.

57. The *slope* of a line is the rise over the run: a change in y divided by a change in x. We write $m = (y_2 - y_1)/(x_2 - x_1)$.

58. The *y-intercept of a line* is that point where the line crosses the y-axis. The usual symbol for it is b.

59. The *equation of a line* is $y = mx + b$, where m is the slope of the line, and b is the y-intercept.

60. *Horizontal lines* look like $y = c$, where c is a constant. *Vertical lines* look like $x = c$, where c is a constant.

61. To *write the equation of a line given its slope and a point on the line*, first write $y = mx + b$, with m written in, since you already know it. The given point looks like (x, y). Plug these two values in for x and y in the equation of the line you have, and solve for b. Finally, rewrite the equation $y = mx + b$ using the newly-found value of b.

62. To *write the equation of a line given two points on the line,* first find the slope of the line using the formula in Item 57 above. Then proceed with Item 61 above. You can check your result by plugging in the unused point.

63. *Parallel lines* have the same slope. *Perpendicular lines* have negative reciprocal slopes: $m_1 = -1/m_2$.

64. *Steps to solve a linear inequality for x:* Follow the steps for solving linear equalities (Item 52 above), except that any time you multiply or divide both sides of the inequality by a negative number, you must reverse the inequality. This does not change the strictness of the inequality.

65. $|a| < b$ is equivalent to $-b < a < b$.

66. To *graph a linear inequality,* first pretend the inequality is an equality and graph that line. Use dashed lines if the original inequality is strict, and solid lines if it is non-strict. Then choose two points, each one clearly on the opposite side of the line from the other. Plug the coordinates for those points into the original inequality. Whichever point produces a true statement will be in the allowed region. Shade in the allowed region.

67. To *solve a system of equations in x and y by substitution,* solve one equation for x, substitute this expression into the other equation, and solve that equation for y. Plug this result back into your solved equation for x, and simply compute its value. Check your solution by plugging both values back into both equations.

68. There are three *elementary operations you can perform on systems of equations*. One is to switch two equations. Another is to multiply an equation by a constant. The third is to add a multiple of one equation to a multiple of the other equation.

69. To *solve a linear system using elimination,* use the elementary operations, particularly the third, to eliminate one variable. Find the other variable, and plug back into an original equation to find the eliminated variable. Check your solution by plugging both values back into both equations.

70. *Systems of linear equations* either have zero solutions (corresponding to parallel, non-intersecting lines), one solution (corresponding to non-parallel lines), or infinitely many solutions (corresponding to identical lines). In three and higher dimensions, you can also have skew lines, which are non-intersecting, non-parallel lines. Skew lines cannot happen in two dimensions.

71. If the bases are repeated, you add the exponents: $a^m \cdot a^n = a^{m+n}$. If the bases are not repeated, you multiply the exponents: $(a^m)^n = a^{m \cdot n}$.

72. *Power of a product:* $(ab)^m = a^m \cdot b^m$. (Exponents distribute over products.)

73. *Power of a sum:* $(a+b)^m \neq a^m + b^m$. (Exponents do **NOT** distribute over sums. See the Binomial Theorem for expanding this expression.)

74. *Quotient of Powers, and Power of Quotients:* $\dfrac{a^m}{a^n} = a^{m-n}$, for $a \neq 0$, and $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$. (Exponents distribute over quotients.)

75. *Zero and Negative Exponents:* $a^0 = 1$, $a^{-n} = \dfrac{1}{a^n}$, $a^n = \dfrac{1}{a^{-n}}$, for $a \neq 0$.

76. *Fractional Exponents:* $a^{1/n} = \sqrt[n]{a}$.

77. *Estimating Sizes of Quantities:* In scientific notation, simply ignore the main number, and do the exponential arithmetic with the powers of ten.

78. An *Exponential Function* is one in which, if the x values are arranged evenly, the y values have a constant ratio, one value to the next. It has the formula $y = ab^x$. If $b > 1$, then the function grows; if $b < 1$, then the function decays. The ratio of one y value to the previous (assuming the change in x is 1) is b.

79. To *multiply polynomials,* you must multiply every term in one polynomial by every term in the other polynomial, and add the results. (Every term by every term.)

80. The *Zero Product Property:* If $ab = 0$, then either $a = 0$ or $b = 0$.

81. A *factor* is a number or quantity that multiplies something else in order to obtain the given number or expression. Factors do not add, subtract, or divide something else in order to obtain the given number or expression.

82. To *Factor a Quadratic* $ax^2 + bx + c$, you:

    a. Factor out the GCF (you always take the smaller of the powers)

    b. Look for a difference of two squares: $x^2 - y^2 = (x-y)(x+y)$.

    c. Look for a perfect square trinomial:

i. The outer two terms must be perfect squares.
ii. The outer two terms must have the same sign. If this sign is negative, factor out the negative sign from all terms before proceeding.
iii. The product of the square roots of the outer terms and 2 must be the middle term.
iv. Then the quadratic factors as $\left(\sqrt{\text{first outer term}} \pm \sqrt{\text{second outer term}}\right)^2$, where the sign is the sign of the middle term.

d. Algorithm:

i. First write $\left(\frac{ax}{}\right)\left(\frac{ax}{}\right)$.
ii. Find the product ac, including sign.
iii. Find the prime factorization of ac using the factor tree.
iv. Find all factor pairs of ac using the factor tree: begin with 1, ac, and increase the 1 according to whether you can get it by a product of numbers in the prime factorization of ac. You are done when the first number has reached $\sqrt{ac}$.
v. Find the factor pair of ac such that the two numbers add to b, including sign. If $ac > 0$, then the two numbers will have the same sign. If $ac < 0$, then the two numbers will have opposite signs. (If this step fails, the quadratic does not factor.) Call these two numbers s and t.
vi. Write $\left(\frac{ax+s}{}\right)\left(\frac{ax+t}{}\right)$.
vii. Divide each of these binomials by its own GCF: $\left(\frac{ax+s}{\text{gcf}(a,s)}\right)\left(\frac{ax+t}{\text{gcf}(a,t)}\right)$. Check that

$$\text{gcf}(a,s) \cdot \text{gcf}(a,t) = a.$$

e. Check your factoring work by multiplying it out, according to Item 79 above, and checking that you get your original quadratic back.

83. The *axis of symmetry* of the general quadratic $ax^2 + bx + c$ is $x = -b/(2a)$. The vertex occurs on this line.

84. You *undo a square* by plus-or-minus square root. That is, to solve $x^2 = d$, you write $x = \pm\sqrt{d}$, if $d > 0$. If $d = 0$, then $x = 0$. If $d < 0$, then the equation has no real solutions.

85. To *complete the square,* you start out with an expression of the form $ax^2 + bx$.

a. You factor out the a thus: $a(x^2 + bx/a)$.

b. You add-and-subtract the same quantity, $\left(\frac{b}{2a}\right)^2$, thus creating a Perfect Square Trinomial. Alternatively, if the original expression is on one side of an *equation*, then you can add the above quantity to both sides. Then you can write

$$a\left(x^2 + \frac{bx}{a} + \left(\frac{b}{2a}\right)^2\right) = a\left(x + \frac{b}{2a}\right)^2,$$

as per the usual Perfect Square Trinomial method.

86. To solve a *quadratic equation* $ax^2 + bx + c = 0$, you can use the *quadratic formula:*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Make sure the equation is in standard form above before using the formula.

87. The *discriminant* of a quadratic equation $ax^2 + bx + c = 0$ is $D = b^2 - 4ac$. It is the thing underneath the square root in the Quadratic Formula. Assuming a, b, and c are real, its sign governs the number of real solutions:

$D > 0$	2 Distinct Real Solutions
$D = 0$	1 Repeated Real Solution
$D < 0$	No Real Solutions: Complex Conjugate Solutions

88. Square Roots, or Radicals, *distribute over multiplication and division:* $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where at least one of a and b is non-negative. Also, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where at least one of a and b is non-negative, and $b \neq 0$.

89. If you *square an equation,* you must ALWAYS check that your solutions satisfy the original equation.

90. The *Distance Formula/Pythagorean Theorem* is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

91. The *Midpoint Formula* is $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

92. You can only *cancel factors* of the *entire* numerator AND the *entire* denominator. Whenever a factor **d**isappears from the **d**enominator, you must **d**isplay a proviso: that factor cannot equal zero.

93. To *multiply two fractions,* you do $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Simplify before multiplying out, and leave result in factored form.

94. To *divide two fractions,* you do $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$. That is, you multiply by the reciprocal of the second fraction. Simplify before multiplying out, and leave result in factored form.

95. To *add or subtract two fractions,* you must:

    a. Determine the LCD.

    b. Multiply each fraction, top and bottom, by what is needed to complete the LCD. (Multiply top and bottom by what is missing.)

    c. Once you have a common denominator, you add or subtract the numerators, and the denominator of the result **is** the LCD. $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$.

    d. Simplify at the end.

Geometry

96. *Undefined terms are* point, line, and plane.

97. *Collinear points* are points that lie on the same line.

98. *Coplanar points* are points that lie in the same plane.

99. The *line segment, or segment* $\overline{AB}$ consists of the endpoints A and B and all points on the line $\overleftrightarrow{AB}$ that are between A and B. Note that $\overline{AB} = \overline{BA}$.

100. The *ray* $\overrightarrow{AB}$ consists of the endpoint A and all points on $\overleftrightarrow{AB}$ that lie on the same side of A as B.

101. If point C lies on $\overleftrightarrow{AB}$ between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are *opposite rays*.

102. Segments and rays are *collinear* if they lie on the same plane.

103. Lines, segments, and rays are coplanar if they lie in the same plane.

104. Two or more geometric figures *intersect* if they have one or more points in common.

105. The *intersection* of two or more geometric figures is the set of points the figures have in common.

106. The *Ruler Postulate:* The points on a line can be matched one-to-one with the real numbers. The real number that corresponds to a point is the *coordinate* of the point.

107. The *Segment Addition Postulate:* If B is between A and C, then $AB + BC = AC$. Conversely, if $AB + BC = AC$, then B is between A and C.

108. Line segments of equal length are called *congruent segments*.

109. The *midpoint* of a segment is the point that divides the segment into two congruent segments.

110. A *segment bisector* is a point, ray, line, line segment, or plane that intersects the segment at its midpoint.

111. An *angle* consists of two different rays with the same endpoint.

112. The two rays making up an angle are the *sides* of the angle.

113. The endpoint of an angle is the *vertex* of the angle.

114. The *angle* with sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be *named* $\angle BAC$.

115. *Protractor Postulate:* Consider $\overleftrightarrow{OB}$ and a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one-to-one with the real numbers from 0 to 180.

116. The *measure* of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$. The measure of $\angle AOB$ is *written* as $m\angle O$.

117. An *acute angle* $\angle A$ has measure $m\angle A < 90°$.

118. A *right angle* $\angle A$ has measure $m\angle A = 90°$.

119. An *obtuse angle* $\angle A$ has measure $90° < m\angle A < 180°$.

120. A *straight angle* $\angle A$ has measure $m\angle A = 180°$.

121. *Angle Addition Postulate:* If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$. That is, $m\angle RST = m\angle RSP + m\angle PST$.

122. *Congruent angles* are angles that have the same measure.

123. An *angle bisector* is a ray that divides an angle into two congruent angles.

124. Two angles are *complementary* if their measure sum is 90°.

125. Two angles are *supplementary* if their measure sum is 180°.

126. *Adjacent angles* are two angles that share a common vertex and side, but have no common interior points.

127. Two adjacent angles are a *linear pair* if their non-common sides are opposite rays.

128. Two angles are *vertical angles* if their sides form two pairs of opposite rays.

129. A *plane figure* is a figure that lies in a plane.

130. A *polygon* is a closed plane figure with the following properties:

    a. It is formed by three or more line segments called *sides*.
    b. Each side intersects exactly sides, one at each endpoint, so that no two sides with a common endpoint are collinear.

131. A polygon is *convex* if no line that contains a side of the polygon contains a point in the interior of the polygon.

132. In an *equilateral* polygon, all sides are congruent.

133. In an *equiangular* polygon, all angles in the interior of the polygon are congruent.

134. A *regular* polygon is a convex polygon that is both equilateral and equiangular.

135. The *perimeter* of a figure is the distance around it.

136. A *circle* is the set of all points in a plane that are the same distance from one point, called its *center*. The common distance each point is from the center is called the *radius* of the circle.

137. The *circumference* of a circle is the distance around it.

138. The *diameter* of a circle is the length of any line segment going through the circle's center with endpoints on the circle.

139. The symbol π is defined as the ratio of a circle's circumference to its diameter. That is, for any circle,

$$\pi = \frac{C}{D}.$$

    π is approximately 3.141592653.

140. The *area* of a figure is the amount of surface covered by it.

141. A *square* is a regular polygon with 4 sides.

142. A *rectangle* is a polygon with 4 sides, and all interior angles being right angles.

143. A *triangle* is a polygon with 3 sides.

144. A side of a triangle is *opposite* to an angle (or *vice versa*) if the side does make up the angle.

145. The *height* or *altitude* of a triangle is the length of the line segment from one vertex to the line containing the opposite side (called the *base*), where this line segment forms right angles with the base.

146. The *perimeter of a square* with side length s is $P = 4s$. The *area of a square* with side length s is $A = s^2$.

147. The *perimeter of a rectangle* with side lengths ℓ and w is $P = 2\ell + 2w$. The *area of a rectangle* with side lengths ℓ and w is $A = \ell w$.

148. The *perimeter of a triangle* with side lengths a, b, and c is $P = a + b + c$. The *area of a triangle* with base b and height h is $A = \frac{1}{2}bh$.

149. The *circumference of a circle* of radius r and diameter $d = 2r$ is $C = \pi d = 2\pi r$. The *area of a circle* of radius r is $A = \pi r^2$.

150. A *conjecture* is an unproven statement that is based on observations.

151. You use *inductive reasoning* when you find a pattern in specific cases and then write a conjecture for the general case.

152. A *counterexample* is a specific case for which a conjecture is false.

153. *Perpendicular lines* are two lines that intersect to form a right angle.

154. *Line Existence Postulate:* Through any two points there exists exactly one line.

155. *Point Existence Postulate (line):* A line contains at least two points.

156. *Intersection of Noncollinear Lines Postulate:* If two non-collinear lines intersect, then their intersection is exactly one point.

157. *Plane Existence Postulate:* Through any three non-collinear points there exists exactly one plane.

158. *Point Existence Postulate (plane):* A plane contains at least three non-collinear points.

159. *Location of Line in a Plane Postulate:* If two points lie in a plane, then the line containing them lies in the plane.

160. *Intersection of Noncoplanar Planes Postulate:* If two non-coplanar planes intersect, then their intersection is a line.

161. A line is a *line perpendicular to a plane* if and only if the line intersects the plane in a point and is perpendicular to every line in the plane that intersects it at that point. We also say the line is *normal* to the plane.

162. A *proof* is a finite sequence of statements, each of which is an axiom, an assumption, or follows from the preceding statements in the sequence by a valid rule of inference. The last statement in the sequence is the *theorem*.

163. *Vertical Angles Congruence Theorem:* Vertical angles are congruent.

164. *Parallel lines* are non-intersecting coplanar lines.

165. *Skew lines* are non-intersecting non-coplanar lines.

166. *Parallel planes* are non-intersecting planes.

167. *Parallel Postulate:* If there is a line and a point not on the line, then there is exactly one line through the point parallel to the given line.

168. *Perpendicular Postulate:* If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line.

169. Two angles are *corresponding angles* if they have corresponding positions.

170. A *transversal* is a line that intersects two or more coplanar lines at different points.

171. Two angles are *alternate interior angles* if they lie between two lines and on opposite sides of the same transversal.

172. *Corresponding Angles Postulate:* If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

173. *Alternate Interior Angles Theorem:* If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

174. *Corresponding Angles Converse Postulate:* If two lines are cut by a transversal so the corresponding angles are congruent, then the lines are parallel.

175. *Alternate Interior Angles Converse Theorem:* If two lines are cut by a transversal so the alternate interior angles are congruent, then the lines are parallel.

176. A *scalene triangle* is a triangle with no congruent sides.

177. An *isosceles triangle* is a triangle with at least two congruent sides.

178. An *equilateral triangle* is a triangle with three congruent sides.

179. An *acute triangle* is a triangle with all angles acute.

180. A *right triangle* is a triangle with one right angle.

181. An *obtuse triangle* is a triangle with one obtuse angle.

182. An *equiangular triangle* is a triangle with three congruent angles.

183. When the sides of a polygon are *extended*, other angles are formed. The original angles are the *interior angles*. The angles that form linear pairs with the interior angles are the *exterior angles*.

184. *Triangle Sum Theorem:* The sum of the measures of the interior angles of a triangle is $180°$.

185. *Exterior Angle Theorem:* The measure of an exterior angle of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

186. In two *congruent figures,* all the parts of one figure are congruent to the *corresponding parts* of the other figure.

187. *Side-Side-Side (SSS) Congruence Postulate:* If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

188. *Side-Angle-Side (SAS) Congruence Postulate:* If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

189. In a right triangle, the sides adjacent to the right angle are called the *legs*, and the side opposite the right angle is called the *hypotenuse*.

190. *Hypotenuse-Leg (HL) Congruence Theorem:* If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

191. *Angle-Side-Angle (ASA) Congruence Postulate:* If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

192. *Angle-Angle-Side (AAS) Congruence Theorem:* If two angles and a non-included side of one triangle are congruent to two angles and the corresponding non-included side of a second triangle, then the two triangles are congruent.

193. *Base Angles Theorem and Converse:* Two sides of a triangle are congruent if and only if the two sides opposite them are congruent.

194. A *median* of a triangle is a segment from a vertex to the midpoint of the opposite side.

195. The three medians of a triangle meet at a point inside the triangle, called the *centroid*.

196. *Triangle Inequality Theorem:* The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

197. The *geometric mean* of two positive numbers a and b is the positive number x that satisfies $\frac{a}{x} = \frac{x}{b}$, and so $x = \sqrt{ab}$.

198. A *scale drawing* is a drawing that is the same shape as the object it represents. The *scale* is a ratio that describes how the dimensions in the drawing are related to the actual dimensions of the object.

199. Two polygons are *similar polygons* if corresponding angles are congruent and corresponding side lengths are proportional.

200. If two polygons are similar, then the ratio of the lengths of two corresponding sides is called the *scale factor*.

201. If two polygons are similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the similar polygons.

202. *Angle-Angle (AA) Similarity Postulate:* If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

203. *Side-Side-Side Similarity Theorem:* If the corresponding side lengths of two triangles are proportional, then the triangles are similar.

204. *Side-Angle-Side (SAS) Similarity Theorem:* If an angle of one triangle is congruent to an angle of a second triangle and the lengths of the sides including these angles are proportional, then the triangles are similar.

205. *Pythagorean Theorem:* In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

206. *Pythagorean Theorem Converse:* If the square of the length of one side is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

207. A *quadrilateral* is a plane figure with four sides.

208. A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel.

209. *Parallelogram Congruence Theorem and Converse:* Opposite sides of a parallelogram are congruent, and opposite angles are as well. Conversely, if opposite sides of a quadrilateral are congruent, or if opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

210. *Parallelogram Diagonal Theorem:* The diagonals of a parallelogram bisect each other.

211. A *rhombus* is a parallelogram with four congruent sides.

212. A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. The parallel sides are the *bases*.

213. A figure in the plane has *line symmetry* if the figure can be mapped onto itself by a reflection in a line, called the *line of symmetry*.

214. A figure in a plane has *rotational symmetry* if the figure can be mapped onto itself by a rotation of 180° or less about the center of the figure, called the *center of symmetry*.

215. A *chord* is a segment whose endpoints are on a circle.

216. A *secant* is a line that intersects a circle in two points.

217. A *cone* has a circular base and a *vertex* that is not in the same plane as the base. The *radius of the cone* is the radius of the base, and the *height of the cone* is the perpendicular distance between the vertex and the base.

218. In a *right cone*, the segment joining the vertex and the center of the base is perpendicular to the base, and the *slant height* is the distance between the vertex and a point on the base edge.

219. A *conic section* is the intersection of a plane with two cones placed tip-to-tip (called double-napped cones).

220. A *tangent* is a line in the plane of a circle that intersects the circle in exactly one point. **This definition applies to conic sections ONLY!!!**

221. For more general curves in the plane, a *tangent* is a line intersecting a curve at a point, which, when zooming in on the intersection, always looks more and more like the curve.

222. *Tangent Perpendicular Theorem:* In a plane, a line is tangent to a circle if and only if the line is perpendicular to a radius of the circle at its endpoint on the circle.

223. *Congruent Tangent Segments Theorem:* Tangent segments from a common external point are congruent.

224. A *central angle* of a circle is an angle whose vertex is the center of the circle.

225. If $m\angle ACB$ of a circle with center C is less than $180°$, then the points on $\odot C$ that lie in the interior of $\angle ACB$ form a *minor arc* with endpoints A and B. The points on $\odot C$ that do not lie on minor arc $\widehat{AB}$ form a *major arc* with endpoints A and B. A *semicircle* is an arc with endpoints that are the endpoints of a diameter.

226. The *measure of a minor arc* is the measure of its central angle.

227. *Arc Addition Postulate:* The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

228. *Arc and Chord Congruence Theorem:* In the same circle, or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

229. An *inscribed angle* is an angle whose vertex is on a circle and whose sides contain chords of the circle. The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the *intercepted arc* of the angle.

230. *Measure of an Inscribed Angle Theorem:* The measure of an inscribed angle is one-half the measure of its intercepted arc.

231. The *standard equation of a circle* with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.

232. The *area of a trapezoid* is product of the height and the average of the base lengths.

233. A *right cylinder* is a solid with congruent bases that lie in parallel planes.

234. The *surface area of a right cylinder* is $S = 2B + Ph$, where B is the area of the base, P is the perimeter of the base, and h is the height of the cylinder.

235. A *pyramid* is a polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex, called the *vertex of the pyramid*.

236. A *regular pyramid* has a regular polygon for a base, and the segment joining the vertex and the center of the base is perpendicular to the base.

237. The *slant height* of a regular pyramid is the height of a lateral face of the regular pyramid.

238. *Surface Area of a Regular Pyramid:* The surface area S of a regular pyramid is $S = B + \frac{1}{2}P\ell$, where B is the area of the base, P is the perimeter of the base, and ℓ is the slant height.

239. *Surface Area of a Right Cone:* The surface area S of a right cone is $S = B + \frac{1}{2}C\ell = \pi r^2 + \pi r\ell$, where B is the area of the base, C is the circumference of the base, r is the radius of the base, and ℓ is the slant height.

240. *Volume of a Cube Postulate:* The volume of a cube is the cube of the length of its side.

241. *Volume Congruence Postulate:* If two polyhedra are congruent, then they have the same volume.

242. *Volume Addition Postulate:* The volume of a solid is the sum of the volumes of all its non-overlapping parts.

243. *Volume of a Cylinder:* The volume V of a cylinder is $V = Bh$, where B is the area of the base, and h is the height.

244. *Cavalieri's Principle:* If two solids have the same height and the same cross-sectional area at every level, then they have the same volume.

245. *Volume of a Pyramid or Cone:* The volume V of a pyramid or cone is $V = \frac{1}{3}Bh$, where B is the area of the base, and h is the height.

246. A *sphere* is the set of all points in space equidistant from a given point, called the *center*. A *radius* of a sphere is a segment from the center to a point on the sphere.

247. A *chord* of a sphere is a segment whose endpoints are on the sphere.

248. A *diameter* of a sphere is a chord that contains the center.

249. *Surface Area of a Sphere:* The surface area S of a sphere is $S = 4\pi r^2$.

250. If a plane contains the center of a sphere, then the intersection is a *great circle* of the sphere.

251. The *circumference of a sphere* is the circumference of one of its great circles.

252. Every great circle of a sphere separates the sphere into two congruent halves called *hemispheres*.

253. *Volume of a Sphere:* The volume V of a sphere is $V = \frac{4}{3}\pi r^3$.

Analytic Geometry

254. *Vertical Line Test:* A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point.

255. *Dependent and Independent Variables:* The input variable is called the *independent variable*, and the output variable is called the *dependent variable*.

256. The *absolute value* $|x|$ of a real number x is defined to be

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}.$$

257. *Transformations of General Graphs:* The graph of $y = a \cdot f(bx - h) + k$ can be obtained from the graph of any function $y = f(x)$ by performing these steps:

    a. Stretch of shrink the graph of $f(x)$ vertically by a factor of $|a|$ if $|a| \neq 1$. If $|a| > 1$, stretch the graph. If $|a| < 1$, shrink the graph.
    b. Stretch or shrink the graph of $f(x)$ horizontally by a factor of $|b|$ if $|b| \neq 1$. If $|b| > 1$, shrink the graph. If $|b| < 1$, stretch the graph.
    c. Reflect the resulting graph from the previous steps in the x-axis if $a < 0$.
    d. Reflect the resulting graph from the previous steps in the y-axis if $b < 0$.
    e. Translate the resulting graph from the previous steps horizontally h units to the right, and k units up.

258. A system of equations with at least one solution is *consistent*; if not, *inconsistent*. A consistent system with exactly one solution is *independent*, otherwise *dependent*.

259. A *matrix* is a rectangular arrangement of numbers in rows and columns.

260. *Addition and Subtraction of Matrices:* To add or subtract two matrices, simply add or subtract the corresponding elements. The matrices must be the same size for this to be valid.

261. *Scalar Multiplication:* To multiply a matrix by a scalar (real number), simply multiply each element in the matrix by the number.

262. *Commutative Property of Matrix Addition:* $A + B = B + A$.

263. *Associative Property of Matrix Addition:* $(A + B) + C = A + (B + C)$.

264. *Distributive Property of Matrix Addition:* $k(A + B) = kA + kB$.

265. *Multiplying Matrices:* To find the element in the ith row and jth column of the product matrix AB, multiply each element in the ith row of A by the corresponding element in the jth column of B, then add the products.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}.$$

The number of columns of A must equal the number of rows of B for the matrix multiplication to be defined.

266. *Non-Commutative Property of Matrix Multiplication:* In general, $AB \neq BA$. When AB does equal BA, we say that A and B *commute*.

267. *Associative Property of Matrix Multiplication:* $A(BC) = (AB)C$.

268. *Left Distributive Property:* $A(B + C) = AB + AC$.

269. *Right Distributive Property:* $(A + B)C = AC + BC$.

270. *Associative Property of Scalar Multiplication:* $k(AB) = (kA)B = A(kB)$.

271. *Determinant of a* 2×2 *Matrix:*

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

272. *Determinant of a* 3×3 *Matrix:*

   a. Repeat the first two columns to the right:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad \rightarrow \quad \left[\begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array}\right].$$

   b. Subtract the sum of the products along one set of diagonals from the sum of the products along the other set of diagonals:

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = (aei + bfg + cdh) - (gec + hfa + idb).$$

273. The *Inverse* of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix},$$

provided $\det(A) \neq 0$.

274. If $Ax = b$, and $\det(A) \neq 0$, then $A^{-1}Ax = A^{-1}b$, or $x = A^{-1}b$.

275. If the denominator of a fraction has an expression of the form $a \pm \sqrt{b}$, where a and b are rational, it is possible to *rationalize the denominator* by multiplying the numerator and denominator by the quantity $a \mp \sqrt{b}$. The denominator will be a rational number.

276. A function has *even symmetry* if $f(x) = f(-x)$ for all x. Graphically, if you reflect the graph of the function about the y-axis, and the result is the same as the original, then the function is even. A function has *odd symmetry* if $f(x) = -f(-x)$ for all x. Graphically, if you rotate the graph of the function about the origin 180 degrees, and the result is the same as the original, then the function is odd.

277. The *identity function* is $f(x) = x$.

278. *Operations on Functions:*

    a. Addition: $(f + g)(x) = f(x) + g(x)$

    b. Subtraction: $(f - g)(x) = f(x) - g(x)$

    c. Multiplication: $(fg)(x) = f(x) \cdot g(x)$

    d. Division: $(f/g)(x) = \dfrac{f(x)}{g(x)}$, whenever $g(x) \neq 0$.

    e. Composition: $(f \circ g)(x) = f(g(x))$. Note that, in general, $f(g(x)) \neq g(f(x))$. That is, function composition is not commutative. It is associative.

279. Two functions f and g are *inverses* of each other if $f(g(x)) = x$ and $g(f(x)) = x$.

280. *Horizontal Line Test:* The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once. We say that f is *one-to-one*.

281. Given a function $y = f(x)$, to *invert the function*, you first swap all the x's and y's, and then solve for y again. The result will be $y = f^{-1}(x)$, read as "y equals f-inverse of x." This is NOT the multiplicative inverse, nor is it the additive inverse.

282. *Definition of i:* $i^2 = -1$, or $i = \sqrt{-1}$.

283. *Complex Conjugate:* Given a complex number $z = a + ib$, the complex conjugate is given by $z^* = a - ib$. In general, to conjugate a complex number, replace all i's with $-i$'s.

284. The *absolute value of a complex number* $z = a + ib$ is given by $|z| = \sqrt{a^2 + b^2}$.

285. *Complex Arithmetic:*

    a. Addition: $(a + ib) + (c + id) = (a + c) + i(b + d)$.

    b. Subtraction: $(a + ib) - (c + id) = (a - c) + i(b - d)$.

c. Multiplication: $(a+ib)(c+id) = ac + iad + ibc - bd = (ac-bd) + i(ad+bc)$.

d. Division:

$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id} \cdot \frac{c-id}{c-id} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \frac{(ac+bd)+i(bc-ad)}{c^2+d^2} = \frac{ac+bd}{c^2+d^2} + i\,\frac{bc-ad}{c^2+d^2}.$$

(Multiply top and bottom by the complex conjugate of the bottom.)

286. *Cubic Factoring Pattern:* $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$; the sign pattern is **S**ame, **O**pposite, **P**ositive (SOP).

287. The *Fundamental Theorem of Algebra:* If $f(x)$ is a polynomial of degree n where $n > 0$, then the equation $f(x) = 0$ has at least one solution in the set of complex numbers.

288. *Descartes' Rule of Signs:* Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ be a polynomial function with real coefficients.

a. The number of *positive real roots (zeros)* of f is equal to the number of sign changes of the coefficients of $f(x)$, or is less than this by an even number.

b. The number of *negative real roots (zeros)* of f is equal to the number of sign changes of the coefficients of $f(-x)$ or is less than this by an even number.

289. To *solve a cubic* $ax^3 + bx^2 + cx + d = 0$, first find all factors of the quotient d/a. Try these as roots of the cubic. If one of them works, say f, then factor out $(x - f)$ by polynomial long division or synthetic division. The result will be $(x - f)$ times a quadratic. Solve the quadratic. This procedure generalizes to higher-order polynomials.

290. The *natural base e* is an irrational number, approximately equal to 2.718281828, but this pattern does not repeat. e^x is the function such that the slope of the tangent line to e^x at any point has a slope equal to e^x - the function's value at that point.

291. The *logarithm function* is the inverse of the exponential function. That is, $\log_a(a^x) = x$, and $a^{\log_a(x)} = x$. If we write $\log_a(x)$, we say, "The log base a of x." The three most important bases are 2, 10, and e, which are written $\log_2(x)$, $\log(x)$, and $\ln(x)$, respectively. The domain of any logarithm function is all real numbers strictly greater than zero.

292. *Properties of Logarithms:*

a. $\log_a(1) = 0$

b. $\log_a(a) = 1$

c. Change of Basis: $\log_a(x) = \dfrac{\log_b(x)}{\log_b(a)}$.

d. Products: $\log_a(xy) = \log_a(x) + \log_a(y)$.

e. Quotients: $\log_a\left(\dfrac{x}{y}\right) = \log_a(x) - \log_a(y)$, provided $y > 0$.

f. Powers: $\log_a(x^y) = y\ \log_a(x)$.

293. *Non-Property of Logarithms:* $\log_a(x+y) \neq log_a(x) + \log_a(y)$.

Trigonometry

294. *Definition of* $\sin(\theta)$*:* $\sin(\theta) = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{\text{Oscar}}{\text{had}}$

295. *Definition of* $\cos(\theta)$*:* $\cos(\theta) = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{\text{a}}{\text{hold}}$

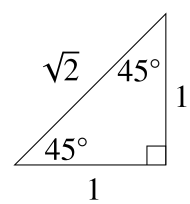
296. *Definition of* $\tan(\theta)$*:* $\tan(\theta) = \dfrac{\text{opp}}{\text{adj}} = \dfrac{\text{on}}{\text{Arthur}} = \dfrac{\sin(\theta)}{\cos(\theta)}$

297. *Definition of* $\csc(\theta)$*:* $\csc(\theta) = \dfrac{1}{\sin(\theta)}$

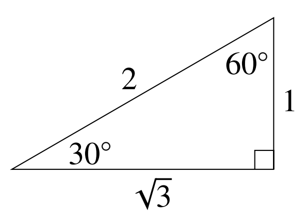
298. *Definition of* $\sec(\theta)$*:* $\sec(\theta) = \dfrac{1}{\cos(\theta)}$

299. *Definition of* $\cot(\theta)$*:* $\cot(\theta) = \dfrac{1}{\tan(\theta)} = \dfrac{\cos(\theta)}{\sin(\theta)}$

300. The $45-45-90$ *triangle:*



301. The $30-60-90$ *triangle:*



302. The *Trigonometric Version of the Pythagorean Theorem:* $\cos^2(\theta) + \sin^2(\theta) = 1$.

303. The *even-ness of* $\cos(\theta)$*:* $\cos(\theta) = \cos(-\theta)$ for all θ.

304. The *odd-ness of* $\sin(\theta)$*:* $\sin(\theta) = -\sin(-\theta)$ for all θ.

305. *Relationship between radians and degrees:* π rad $= 180°$.

306. The *addition of angles formula for* sin*:* $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$.

307. The *addition of angles formula for* cos*:* $\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

308. The graphs of the sin and cos functions:

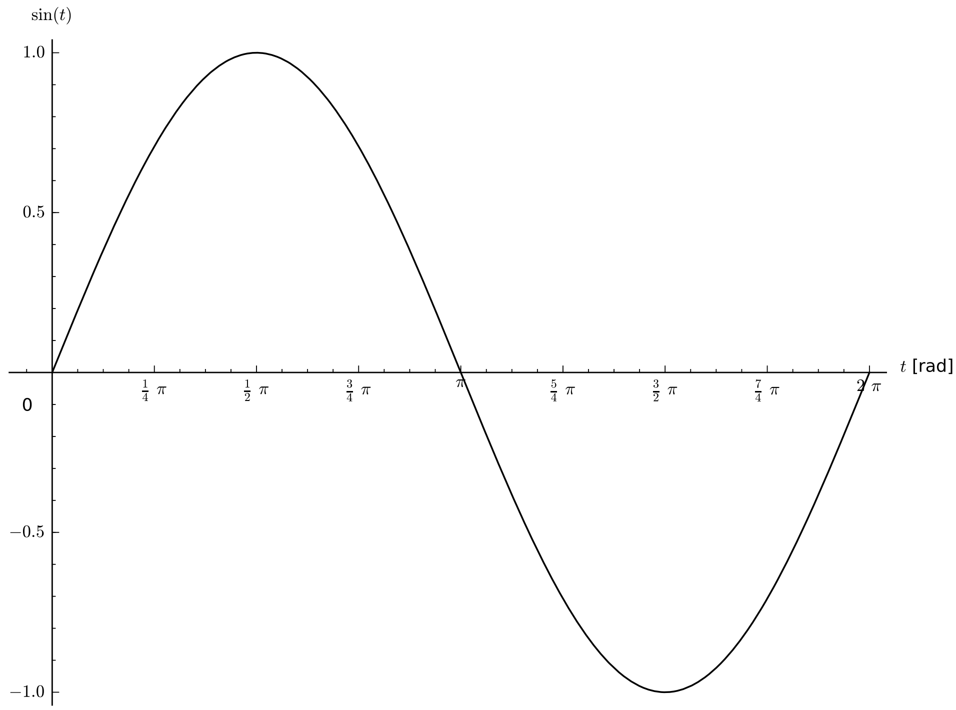


Figure 1: sin(t) versus t

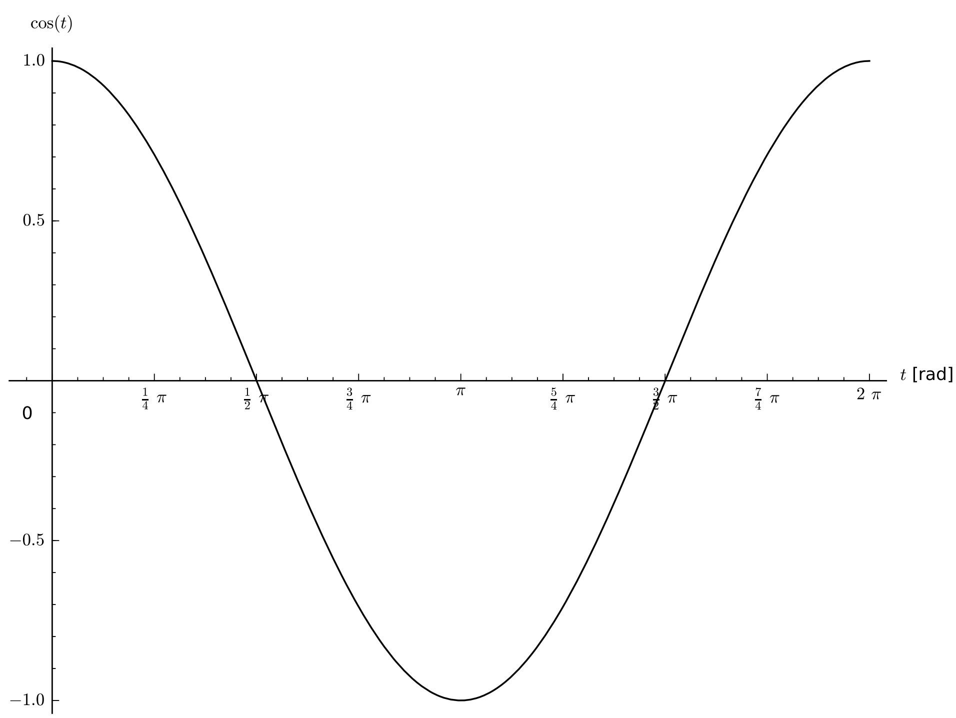


Figure 2: cos(t) versus t

Probability and Statistics

309. An *individual* is a person or object that we are studying, or in which we are interested.

310. A *population* is all the individuals that we are studying, or in which we are interested.

311. A *parameter* is a number that describes the population.

312. A *sample* is a group of individuals chosen from the population.

313. A *statistic* is a number that describes a sample.

314. A *sampling distribution* results when you take all possible samples from the population of the same size, and compute the same statistic on each sample.

315. A *simple random sample,* or SRS, is a sample chosen from the population in such a way that any other sample of the same size would have been equally likely to be chosen.

316. *Categorical Data* is data for which it does not make sense to take an average.

317. *Quantitative Data* is data for which it does make sense to take an average.

318. In an *observational study,* the researcher merely observes what is happening or what has happened in the past and tries to draw conclusions based on these observations. Cause and effect relationships are impossible to determine from an observational study.

319. In an *experiment,* the researcher manipulates one of the variables and tries to determine how the manipulation influences other variables. Cause and effect relationships are possible to determine from an experiment, depending on how the researcher conducts the experiment.

320. A *confounding variable* is one that influences the dependent or outcome variable but was not separated from the independent variables.

321. A *frequency distribution* is the organization of raw data in table form, using classes and frequencies.

322. A *histogram* is a graph that displays single-variable data by using contiguous vertical bars of various heights to represent the frequencies of the classes.

323. A *stem-and-leaf plot* is a data plot that uses part of the data value as the stem and part of the data value as the leaf to form groups or classes.

324. A *scatter plot* is a graph of ordered pairs of data values (two-variable data) that is used to determine if a relationship exists between the two variables.

325. Single-variable data is *right-skewed* if the majority of the data points fall to the left of the mean, and cluster at the lower end of the distribution. Single-variable data is *left-skewed* if the majority of the data points fall to the right of the mean, and cluster at the higher end of the distribution. Single-variable data is *symmetric* if it is not right-skewed or left-skewed.

326. The *mean* is the sum of the data values, divided by the number of data values:

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_N}{N},$$

where $\overline{x}$ is the sample mean, n is the sample size, μ is the population mean, and N is the population size. The mean is *sensitive to outliers*, and is a good measure of central tendency when the distribution is roughly symmetric with no outliers.

327. The *median* is the midpoint of the data. To compute the median,

    a. Arrange the data in order.
    b. If there are an odd number of data points, the median is the middle data point.
    c. If there are an even number of data points, the median is the average of the two middle data points.

    The median is *not sensitive to outliers,* and is a better measure of central tendency if the data is skewed or has outliers.

328. *Mean-Median Criterion for Skewness:* If the mean is significantly higher than the median, the distribution is right-skewed; if the mean is significantly lower than the median, the distribution is left-skewed. If the mean is approximately equal to the median, the distribution is roughly symmetric.

329. The *variance* is the average of the squares of the distance each value is from the mean:

$$\sigma^2 = \frac{\sum(X-\mu)^2}{N}, \qquad \sigma = \sqrt{\sigma^2} = \sqrt{\frac{\sum(X-\mu)^2}{N}},$$

    for populations, and

$$s^2 = \frac{\sum(X-\overline{x})^2}{n-1}, \qquad s = \sqrt{s^2} = \sqrt{\frac{\sum(X-\overline{x})^2}{n-1}},$$

    for samples.

330. The *Range Rule of Thumb* is that $s \approx (\max - \min)/4$.

331. The *Empirical Rule* says that if a distribution is roughly normal, or bell-shaped, then approximately:

    a. 68% of the data values fall within 1 standard deviation from the mean,
    b. 95% of the data values fall within 2 standard deviations from the mean, and
    c. 99.7% of the data values fall within 3 standard deviations from the mean.

332. A *percentile* is a number that describes what percent of the data lies below a given value.

333. The *first quartile,* denoted Q_1, is the median of the data values below the median (also the 25th percentile), and the *third quartile,* denoted Q_3, is the median of the data values above the median (also the 75th percentile).

334. The *five-number summary* is the set of numbers $[\min,\ Q_1,\ \text{median},\ Q_3,\ \max]$.

335. A *box-plot* is a graph of a data set obtained by drawing a horizontal line from the minimum data value to Q_1, drawing a horizontal line from Q_3 to the maximum data value, and drawing a box whose vertical sides pass through Q_1 and Q_3 with a vertical line inside the box passing through the median.

336. The *interquartile range,* denoted IQR, is defined by $\text{IQR} = Q_3 - Q_1$.

337. An *outlier* is defined as any data point that is above $Q_3 + 1.5\,\text{IQR}$, or below $Q_1 - 1.5\,\text{IQR}$.

338. For any data point x, its z score is defined by $z = (x - \mu)/\sigma$, for populations, or $z = (x - \overline{x})/s$, for samples.

339. To *describe one-variable data*, use the acronym **SOCS**, which is

   a. **S**hape: skewness, number of modes, gaps

   b. **O**utliers: see Number 337 above

   c. **C**enter: mean and median

   d. **S**pread: standard deviation and IQR.

340. A *probability experiment* is a chance process that leads to well-defined results called outcomes.

341. A *sample space* is the set of all possible outcomes of a probability experiment.

342. *Formula for Classical Probability:* The probability of any event E is the number of outcomes in E divided by the number of outcomes in the sample space.

343. The *complement of an event* E is the set of outcomes in the sample space that are not included in the outcomes of event E; this is denoted $\overline{E}$.

344. *Probability Rules:*

   a. The probability of any event E, denoted by $P(E)$, satisfies the inequality $0 \leq P(E) \leq 1$.

   b. If an event E cannot occur, then $P(E) = 0$.

   c. If an event E is certain, then $P(E) = 1$.

   d. The sum of all the probabilities of all the outcomes in the sample space is 1.

   e. $P(E) + P(\overline{E}) = 1$.

345. Two events are *mutually exclusive events* if they cannot occur at the same time (i.e., they have no outcomes in common).

346. *Addition Rule:* $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. If A and B are mutually exclusive, then $P(A \text{ and } B) = 0$. Here the word 'or' is taken in its inclusive sense.

347. Two events A and B are *independent events* if the fact that A occurs does not affect the probability of B occurring. Events that are not independent are dependent.

348. The *conditional probability* of an event B occurring given that event A has already occurred is denoted $P(B|A)$, and is given by $P(B|A) = \dfrac{P(B \text{ and } A)}{P(A)}$.

349. *Multiplication Rule:* The probability of two events occurring is $P(A \text{ and } B) = P(A) \cdot P(B|A)$.

350. *Criterion for Independence:* Two events A and B are independent if and only if any of the following is true:

   a. $P(B|A) = P(B)$

   b. $P(A|B) = P(A)$

   c. $P(A \text{ and } B) = P(A) \cdot P(B)$.

351. *Fundamental Counting Rule:* In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 possibilities and the third has k_3, and so forth, the total number of possibilities of the sequence will be $k_1 \cdot k_2 \cdot k_3 \cdots k_n$.

352. For any counting number n, we define the *factorial* to be $n! = n(n-1)(n-2)\cdots 1$, and $0! = 1$.

353. A *permutation* is an arrangement of n objects in a specific order.

354. *Permutation Rule:* The arrangement of n objects in a specific order using r objects at a time is called a permutation of n objects taking r objects at a time. It is written as ${}_nP_r$, and the formula is

$${}_nP_r = \frac{n!}{(n-r)!}.$$

We say "n permute r".

355. A *combination* is a selection of distinct objects without regard to order.

356. *Combination Rule:* The number of combinations of r objects selected from n objects is denoted by ${}_nC_r$ and is given by the formula

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

We say "n choose r".

357. A *random variable* is a variable whose values are determined by chance.

358. A *discrete probability distribution* consists of the values a random variable can assume and the corresponding probabilities of the values.

359. The *mean of a discrete random variable* is $\mu = \sum X \cdot P(X)$. This is also called the *expected value*. The *standard deviation of a discrete random variable* is $\sigma = \sqrt{\sum[(X-\mu)^2 P(X)]}$.

360. A *binomial distribution* is a probability experiment that satisfies the following four requirements:

   a. **B**inary: each trial has either a "success" or a "failure".
   b. **I**ndependent: The outcomes of each trial must be independent from one another.
   c. **N**umber of trials must be fixed in advance.
   d. **S**uccess probability must be the same from one trial to the next.

   The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a *binomial distribution*.

361. *Binomial Probability Formula:* In a binomial experiment, the probability of exactly X successes in n trials is

$$P(X) = \frac{n!}{(n-X)!\,X!} \cdot p^X \cdot q^{n-X},$$

where p is the probability of success, $q = 1 - p$ is the probability of failure, n is the number of trials, and X is the number of successes in n trials.

362. The *mean and standard deviation for a binomial distribution* are $\mu = np$ and $\sigma = \sqrt{npq}$.

363. A *normal distribution* is a continuous, symmetric, bell-shaped distribution of a variable.

364. The *standard normal distribution* has mean 0 and standard deviation 1.

365. The *Central Limit Theorem:* As the sample size n increases without limit, the shape of the distribution of the sample means taken with replacement from a population with mean μ and standard deviation σ will approach a normal distribution with mean μ and a standard deviation $\sigma/\sqrt{n}$.

366. A *point estimate* is a specific numerical value estimate of a parameter.

367. The *confidence level* of an interval estimate of a parameter is the proportion of estimation intervals constructed in the same way that contain the parameter.

368. A *confidence interval* is a specific interval estimate of a parameter determined by using data obtained from a sample and by using the specific confidence level of the estimate. A typical confidence interval looks like $[\overline{x} - zs, \overline{x} + zs]$.

369. The *interpretation of a confidence interval* is that "we are CL confident that the interval captures the parameter value." Here CL is the confidence level, as a percent.

370. If the population standard deviation σ is known, then you construct confidence intervals for the mean by using the standard normal distribution. If σ is not known, you must use the t distribution, with the degrees of freedom equal to the sample size minus one. That is, d.f. $= n - 1$. In both cases, the sample must be a simple random sample, and either the population is normally distributed, or the sample size is greater than 33, or both.

371. To construct a confidence interval for a proportion, you must have $n\hat{p} \geq 10$ and $n\hat{q} \geq 10$. Then you may use the standard normal distribution to construct the confidence interval, which will be $\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$. This assumes the sample is a simple random sample, and that the BINS criteria for a binomial experiment are satisfied (Item 360) above.

372. Hypothesis testing, while used frequently, is starting to be criticized more and more. A confidence interval gives all the same information that an hypothesis test gives, and more besides. You should default to a confidence interval.

373. A *statistical hypothesis* is a conjecture about a population parameter. This conjecture may or may not be true.

374. The *null hypothesis,* symbolized by H_0, is a statistical hypothesis that states that there is no difference between a parameter and a specific value, or that there is no difference between two parameters. The "null" hypothesis is the "dull" hypothesis.

     The *alternative hypothesis,* symbolized by H_1 or H_a, is a statistical hypothesis that states the existence of a difference between a parameter and a specific value, or states that there is a difference between two parameters.

375. A *statistical test* uses the data obtained from a sample to make a decision about whether the null hypothesis should be rejected. The only two outcomes of a statistical test are that we find enough evidence to reject the null hypothesis, or we do not find enough evidence to reject the null hypothesis. That is, we either reject the null, or fail to reject the null. We never affirm the null hypothesis.

376. A *Type I* error occurs if you reject the null hypothesis when it is true. A *Type II* error occurs if you do not reject the null hypothesis when it is false. To remember which is which, think of the boy who cried wolf: the *first* error the villagers committed was a Type I error (they thought there was a wolf when there wasn't), and the *second* error the villagers committed was a Type II error (they thought there wasn't a wolf when there was).

377. The *significance level* α is the maximum probability of committing a Type I error.

378. A *one-tailed test* indicates that the null hypothesis should be rejected when the test value is in the critical region on one side of the mean. A *two-tailed test* indicates that the null hypothesis should be rejected when the test value is in either of the two critical regions. The two-tailed test is more rigorous, and unless you have strong reason to suspect that the parameter is only on one side of the test value, you should select the two-tailed test.

379. *Steps for Testing an Hypothesis:*

    a. State the hypothesis and identify the claim.

    b. Find the critical value.

    c. Compute the test value.

    d. Make the decision to reject or fail to reject the null hypothesis.

    e. Summarize the results.

380. The *z-test for the mean* can be used if $n \geq 33$ or when the population is normally distributed and σ is known. The formula for the z-test is
$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}},$$
where $\overline{x}$ is the sample mean, μ is the hypothesized population mean, σ is the population standard deviation, and n is the sample size. This assumes the sample is a simple random sample.

381. The *t-test for the mean* can be used if $n \geq 33$ or when the population is normally distributed and σ is not known. The formula for the t-test is
$$t = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}},$$
where $\overline{x}$ is the sample mean, μ is the hypothesized population mean, σ is the population standard deviation, and n is the sample size. The degrees of freedom are d.f. $= n - 1$. This assumes the sample is a simple random sample.

382. The *z-test for the proportion* can be used if $n \geq 33$ or when the population is normally distributed. The formula for the z-test is
$$z = \frac{\hat{p} - p}{\sqrt{pq/n}},$$
where $\hat{p}$ is the sample proportion, p is the hypothesized population proportion, and n is the sample size. This assumes the sample is a simple random sample, and that $np \geq 10$ and $nq \geq 10$.

383. The *correlation coefficient* computed from the sample data measures the strength and direction of a linear relationship between two quantitative variables. The symbol for the sample correlation coefficient is r. The *coefficient of determination* is the square of the correlation coefficient, and is denoted r^2, or sometimes R^2. We interpret the coefficient of determination as, "R^2% of the variation in the dependent variable is explained by the linear fit."

384. To *describe two-variable data*, use the acronym **DOFS**, which is

   a. **D**irection: positive or negative
   b. **O**utliers or influential points
   c. **F**orm: linear, power, exponential, logarithmic, polynomial, etc.
   d. **S**trength: Use the coefficient of determination (See 383 above.)

385. Correlation does not imply causation.

   a. There could be a forward cause-and-effect relationship.
   b. There could be an inverse cause-and-effect relationship.
   c. A third variable could cause the relationship.
   d. There may be a complexity of relationships between the variables.
   e. The relationship may be coincidental.

386. *Formula for the Chi-Square Goodness-of-Fit Test:*

$$\chi^2 = \sum \frac{(O - E)^2}{E},$$

where O is the observed frequency, and E is the expected frequency. This assumes a SRS, and that each expected frequency is 5 or more.