The result for the special case of $k_1 = k_2 = k_3 = k$ and $m_1 = m_2 = m$ becomes

$$\begin{aligned}
\dot{x}_1(t) &= x_3(t), \\
\dot{x}_2(t) &= x_4(t), \\
\dot{x}_3(t) &= -\frac{2k}{m}x_1(t) + \frac{k}{m}x_2(t), \\
\dot{x}_4(t) &= \frac{k}{m}x_1(t) - \frac{2k}{m}x_2(t).
\end{aligned} \tag{5.94}$$

Thus, the form of the last equation is $\dot{\mathbf{x}} = A\mathbf{x}$, where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{2k}{m} & \dfrac{k}{m} & 0 & 0 \\ \dfrac{k}{m} & -\dfrac{2k}{m} & 0 & 0 \end{bmatrix}$$

Analysis. Letting $k = 1$ and $m = 1$ for simplicity, the eigenvalues of the system matrix were found using the MATLAB command **eig** to be $\pm 1.0i$ and $\pm\sqrt{3}i$. The fundamental, or *natural*, frequencies of oscillation are $\omega_1 = 1$ and $\omega_2 = \sqrt{3}$ radians per second. Examining the eigenvectors (not shown) indicates that there are two modes of oscillation, as you are asked to investigate in Problem 5.19. The eigenvectors associated with the natural frequencies are called the *normal modes* of the system. They describe the relative amplitudes of the displacement of the masses if the system vibrated only at the corresponding natural frequency. However, the absolute amplitudes depend on the initial conditions.

Test cases. The script EX5_18.M solves the equations by calling M-function `cldesf` to define these equations for **ode23**.

MATLAB Script

```
Example 5.18
% EX5_18.M  Use MATLAB ode23 to solve the system
%  y1''=-2y1+ y2
%  y2''=  y1-2y2
%   transformed into the system xdot=Ax where A is 4x4
% INPUTS: Initial time, final time, initial conditions and title
% OUTPUT: A (global variable); Plot of motion y1(t), y2(t)
% Pass A to function CLDESF
A=[0 0 1 0;0 0 0 1;-2 1 0 0;1 -2 0 0]      % System matrix
t0=input('Initial time=  ')
tf=input('Final time=  ')
x0=input('[y1(t0) y2(t0) doty1(t0) doty2(t0)] =  ')
x0t=x0';                % Transpose of initial conditions for ode23
% Calls function cldesf to define state equations.
[t,x]=ode23('cldesf',[t0,tf],x0t,[],A);  % Numerical solution of system
% y values
y1=x(:,1);              % Change to physical variables in example
y2=x(:,2);
% Plot y1 and y2, the motion of the masses
```