Suppose that U is an orthogonal matrix. Show that any eigenvalue λ of U satisfies $|\lambda| = 1$.

★**2.26** You saw in Example 2.8 that the orthogonal matrix U is of the form

$$U = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$

Certainly, the rows (and columns) of U are orthogonal to each other.

(a) What additional condition on a, b must be satisfied so that U is an orthogonal matrix?

(b) Use part (a) and your knowledge of trigonometry to write a, b in terms of an angle θ.

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(c) Using part (b), write down the general form of all 2×2 orthogonal matrix for a given angle θ.

(d) Construct 2×2 orthogonal matrix U that rotates the vectors in C from Example 2.8 $\frac{\pi}{3}$ radians counterclockwise.

(e) Construct 2×2 orthogonal matrix U that rotates the vectors in C from Example 2.8 $\frac{\pi}{2}$ radians counterclockwise.

(f) Write down the general form of $U^{-1} = U^T$ for the orthogonal matrix in part (c). Describe in words what U^T does to vectors in C if U rotates the vectors in C by θ radians.

★**2.27** Let n be an even positive integer, and let $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

(a) Find a matrix H so that

$$H\mathbf{v} = \frac{1}{2}\begin{bmatrix} v_1 + v_2 \\ v_3 + v_4 \\ \vdots \\ v_{n-1} + v_n \end{bmatrix}$$

(b) Note that the rows of H are orthogonal to each other. However, any inner product of a row with itself does not result in the value 1. Can you create a matrix $\tilde{H}$ that has the same structure (i. e., the placement of zeros is the same, so that the rows remain orthogonal, and the nonzero numbers are the same but not equal to $\frac{1}{2}$), so that the rows of $\tilde{H}$ have unit length?